Towards a flexible statistical modelling by latent factors for evaluation of simulated responses to climate forcings: Part III (Supplement)

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1 Analysis of the autocorrelation structure

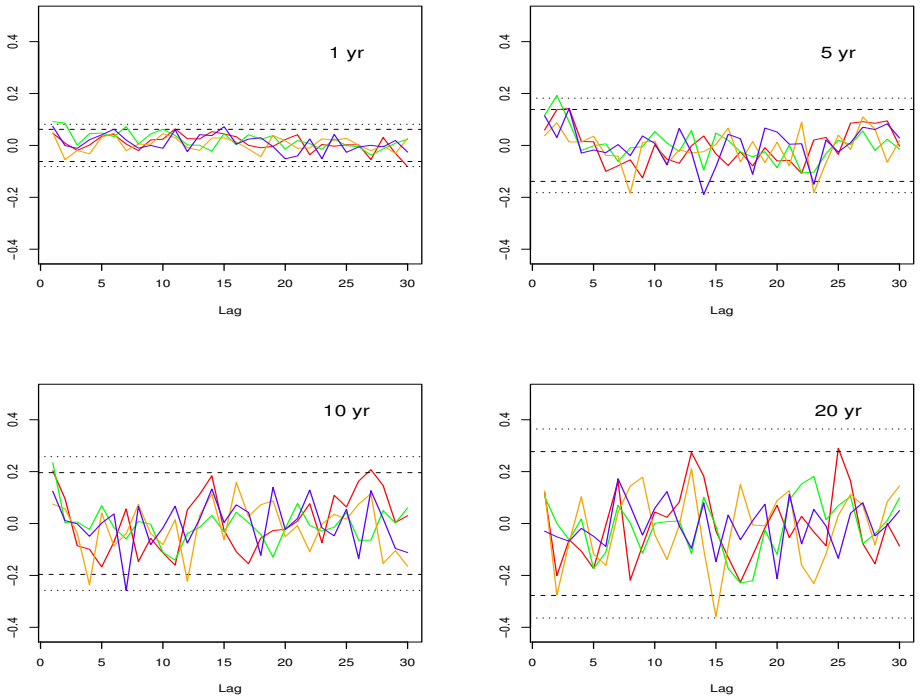

Figure S1.1. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Sol repl}\,i\,t}\} = \{x_{\text{Sol repl.i}\,t} - \bar{x}_{\text{Sol}\,.t}\}$- sequences, $i = 1, 2, 3, 4$ (Europe). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

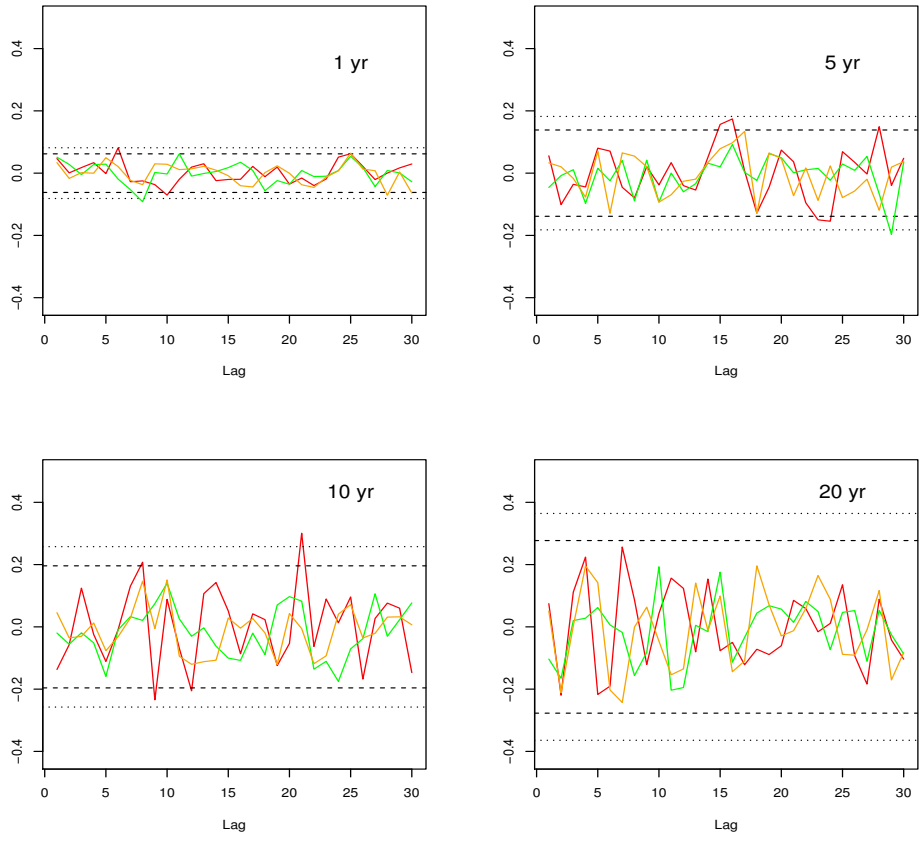

Figure S1.2. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Orb repl}\,i\,t}\} = \{x_{\text{Orb repl}.i\ t} - \bar{x}_{\text{Orb}\,.t}\}$- sequences, $i = 1, 2, 3$ (Europe). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

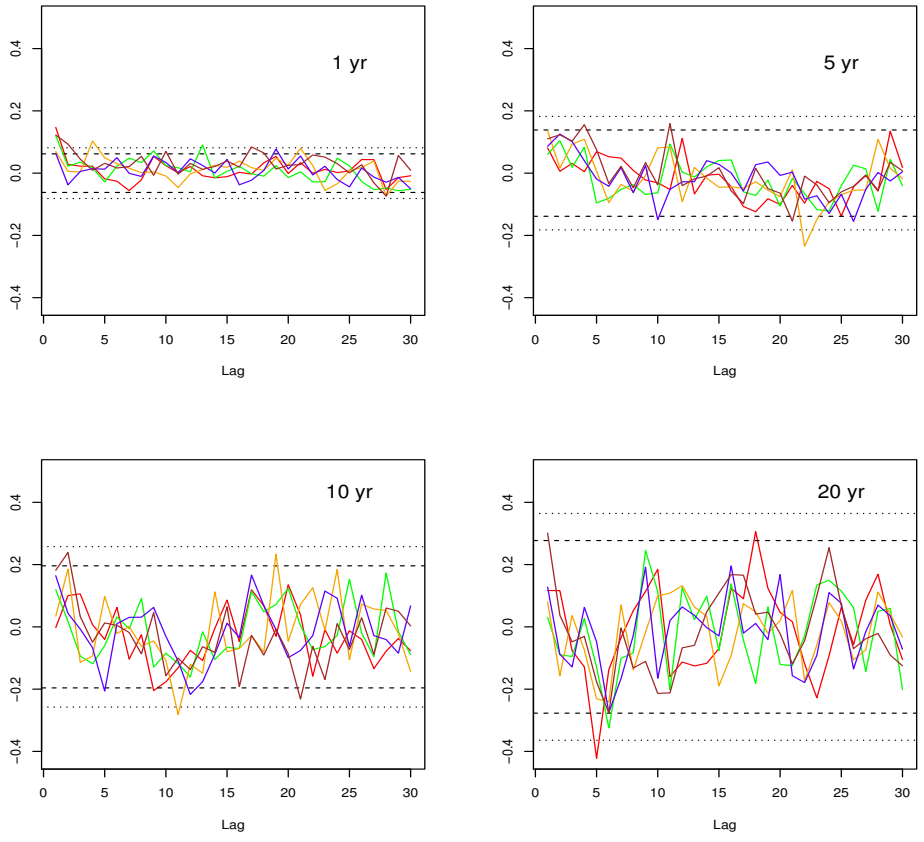

Figure S1.3. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Volc repl}\,i\,t}\} = \{x_{\text{Volc repl.i }t} - \bar{x}_{\text{Volc .}t}\}$- sequences, $i = 1, 2, \ldots, 5$ (Europe). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

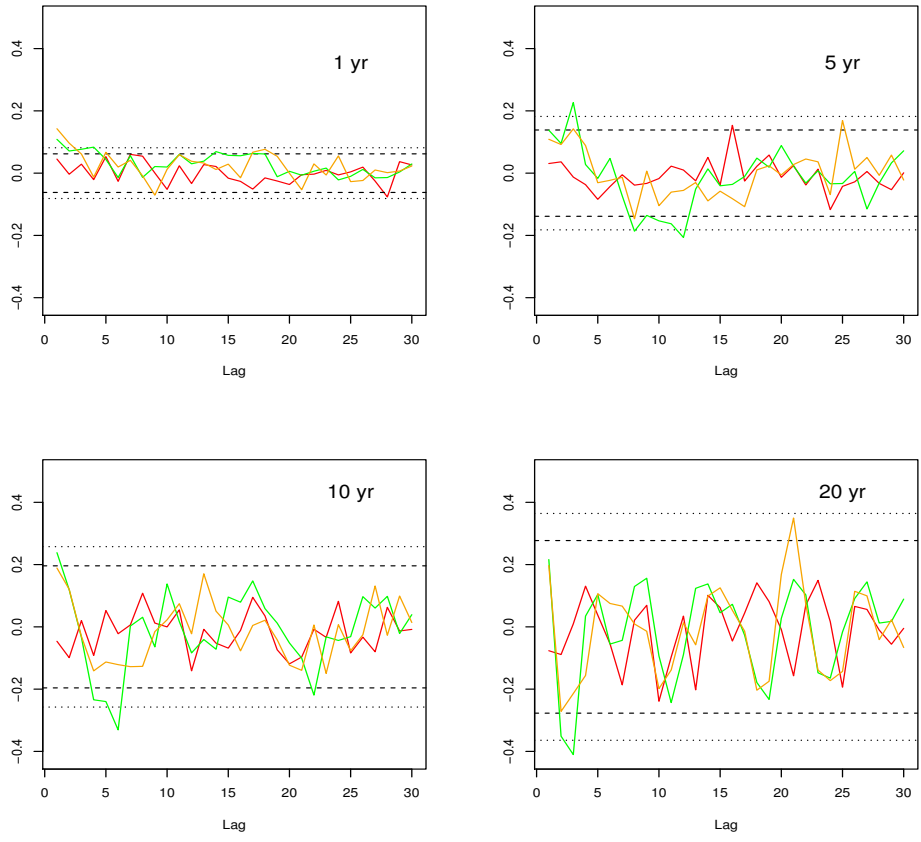

Figure S1.4. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Land repl}\,it}\} = \{x_{\text{Land repl.i}\;t} - \bar{x}_{\text{Land}\;.t}\}$- sequences, $i = 1, 2, 3$ (Europe). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1,\ 5,\ 10,\ 20$.

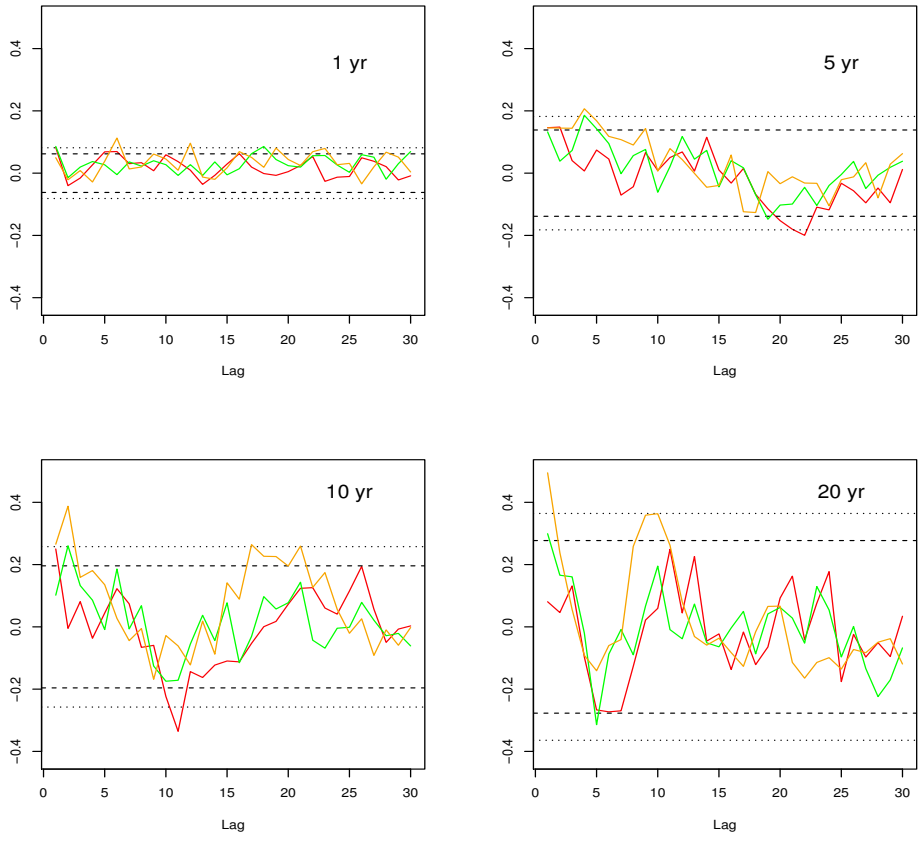

Figure S1.5. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Ghg repl.i}\,t}\} = \{x_{\text{Ghg repl.i}\,t} - \bar{x}_{\text{Ghg}\,.t}\}$- sequences, $i = 1, 2, 3$ (Europe). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

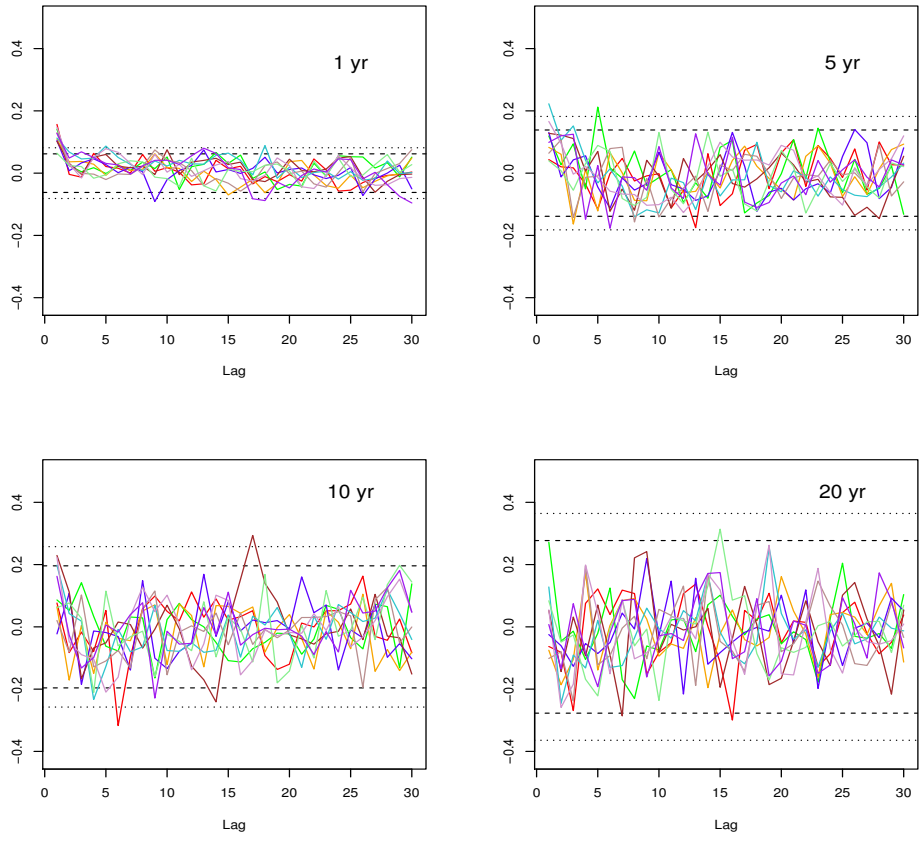

Figure S1.6. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{comb repl}\,it}\} = \{x_{\text{comb repl.i}\,t} - \bar{x}_{\text{comb}\,.t}\}$- sequences, $i = 1, 2, \ldots, 10$ (Europe). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

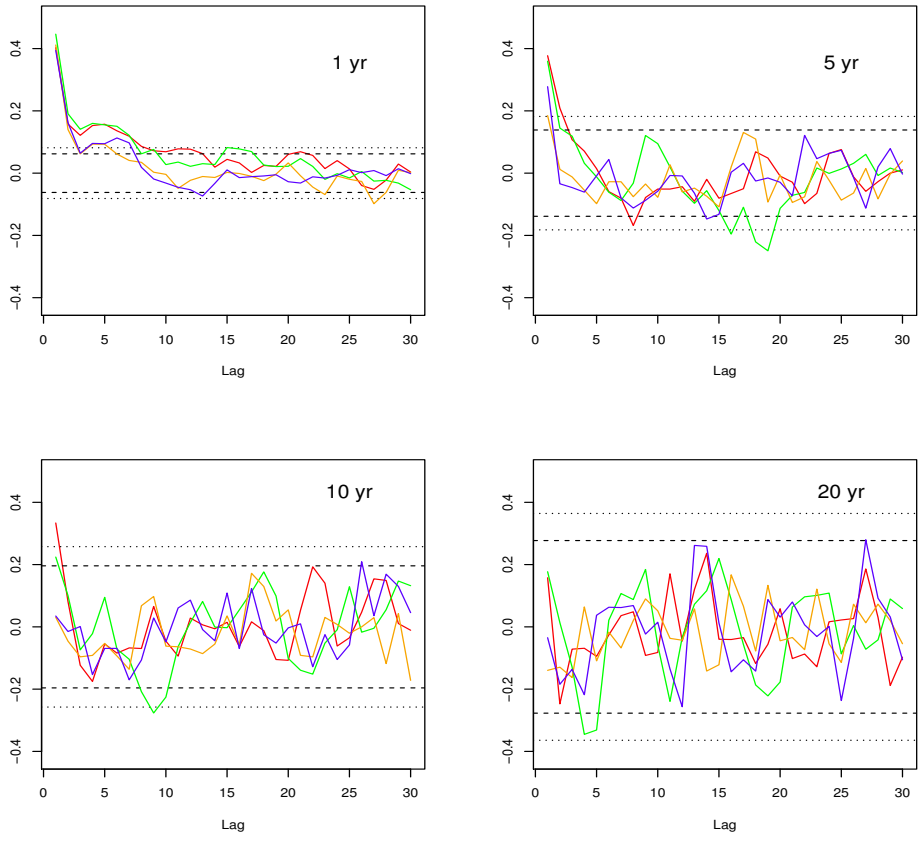

Figure S1.7. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Sol repl.i } t}\} = \{x_{\text{Sol repl.i } t} - \bar{x}_{\text{Sol .} t}\}$- sequences, $i = 1, 2, 3, 4$ (Arctic). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

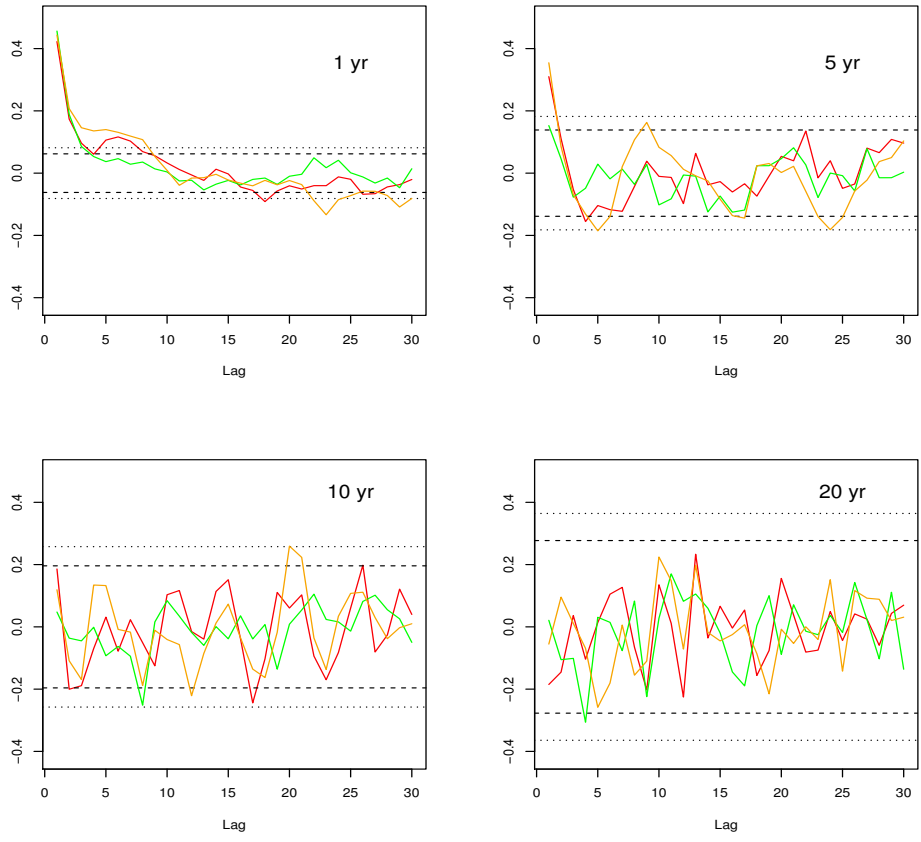

Figure S1.8. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Orb repl i } t}\} = \{x_{\text{Orb repl.i } t} - \bar{x}_{\text{Orb .}t}\}$- sequences, $i = 1, 2, 3$ (the Arctic). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

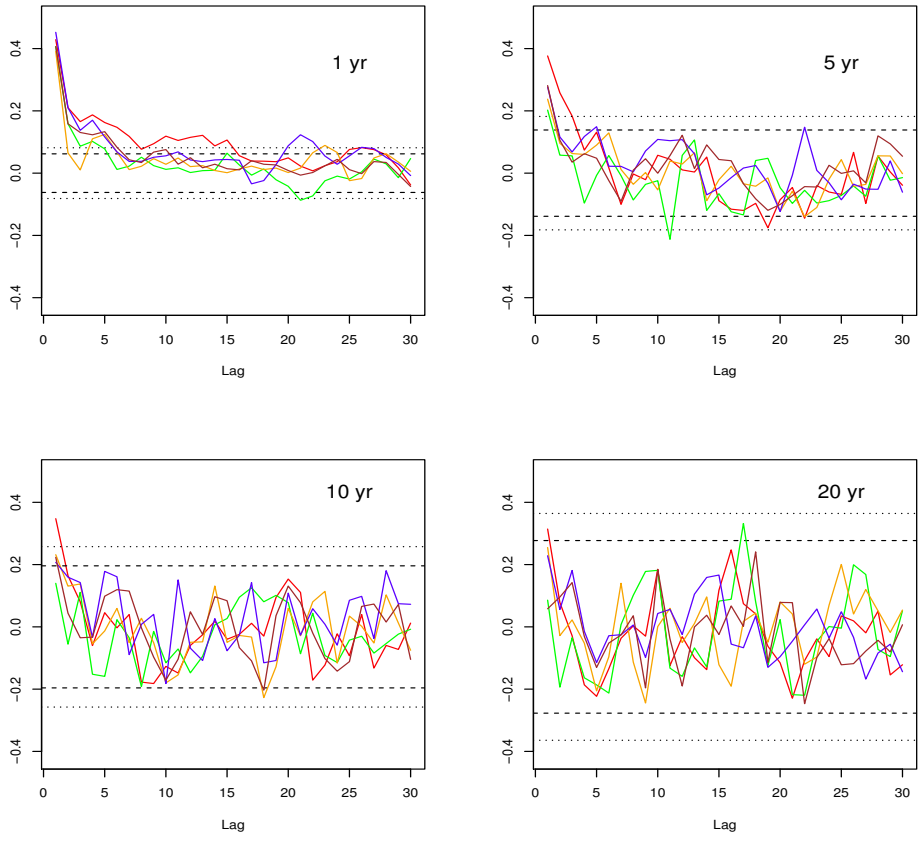

Figure S1.9. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Volc repl}\,i\,t}\} = \{x_{\text{Volc repl.i}\ t} - \bar{x}_{\text{Volc}\ .t}\}$- sequences, $i = 1, 2, \ldots, 5$ (the Arctic). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

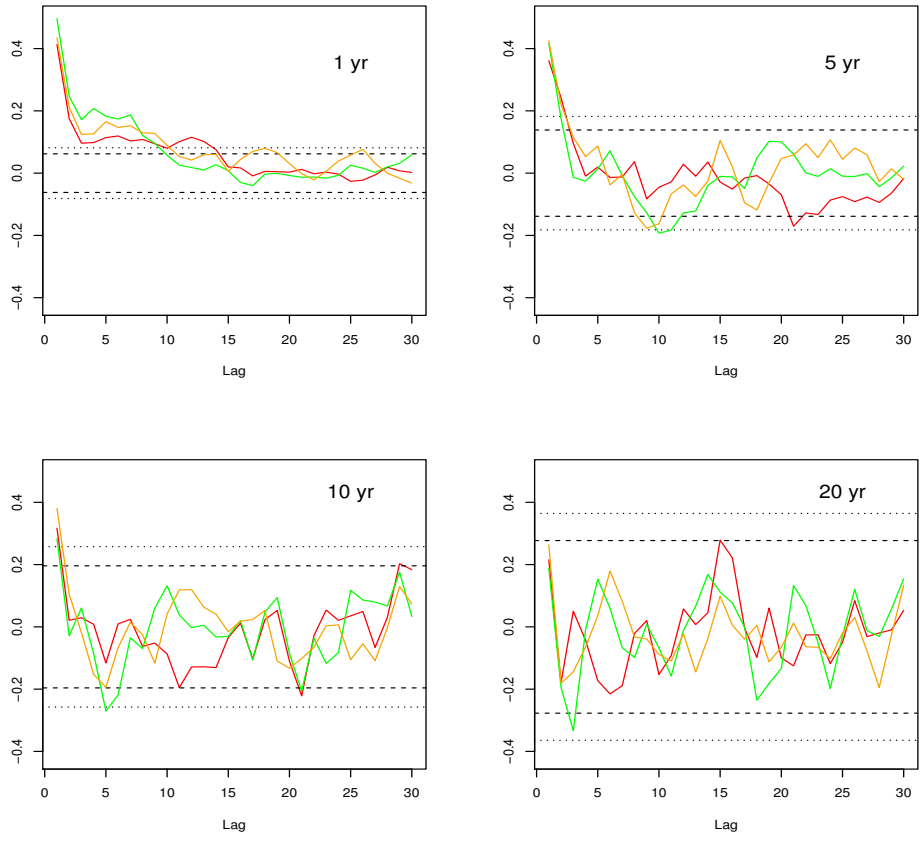

Figure S1.10. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Land repl}\,it}\} = \{x_{\text{Land repl.i}\,t} - \bar{x}_{\text{Land}\,.t}\}$- sequences, $i = 1, 2, 3$ (the Arctic). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

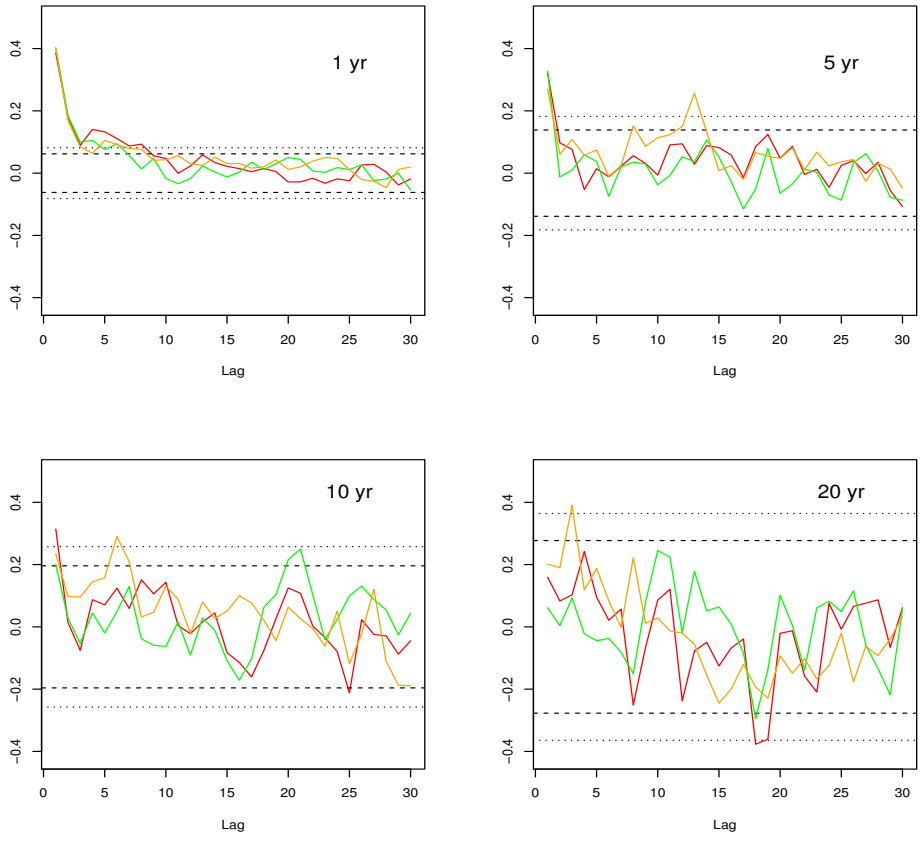

Figure S1.11. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Ghg repl.i}\,t}\} = \{x_{\text{Ghg repl.i}\,t} - \bar{x}_{\text{Ghg}\,.t}\}$- sequences, $i = 1, 2, 3$ (the Arctic). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

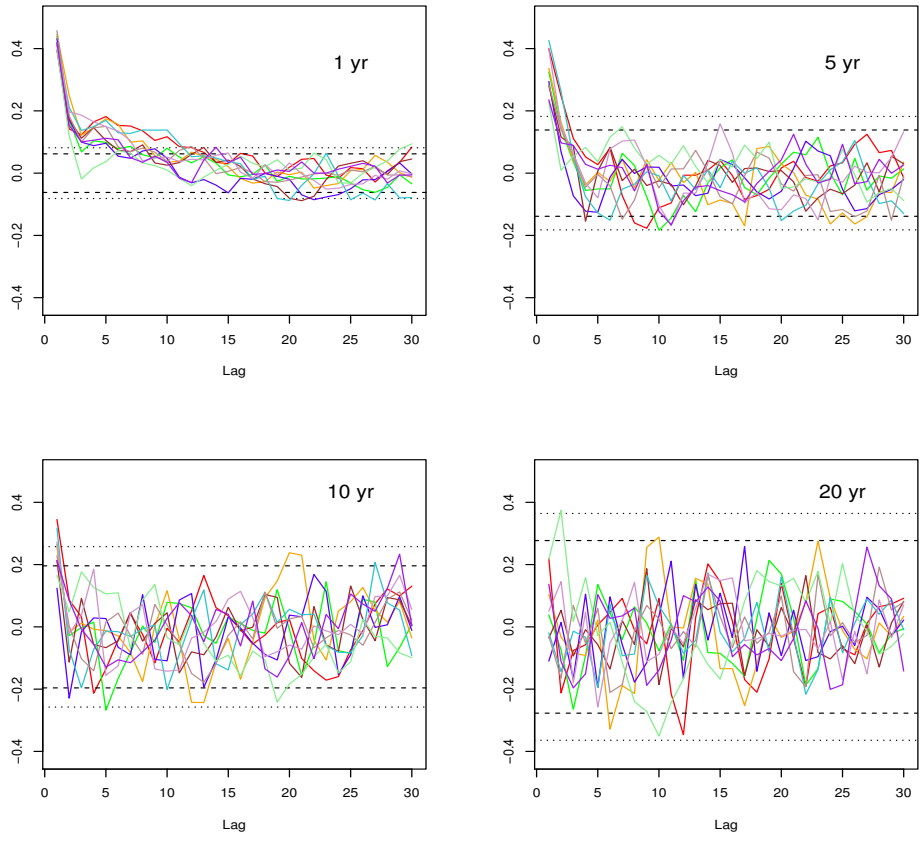

Figure S1.12. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{comb repl}\,it}\} = \{x_{\text{comb repl.i}\;t} - \bar{x}_{\text{comb}\;.t}\}$- sequences, $i = 1, 2, \ldots, 10$ (the Arctic). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

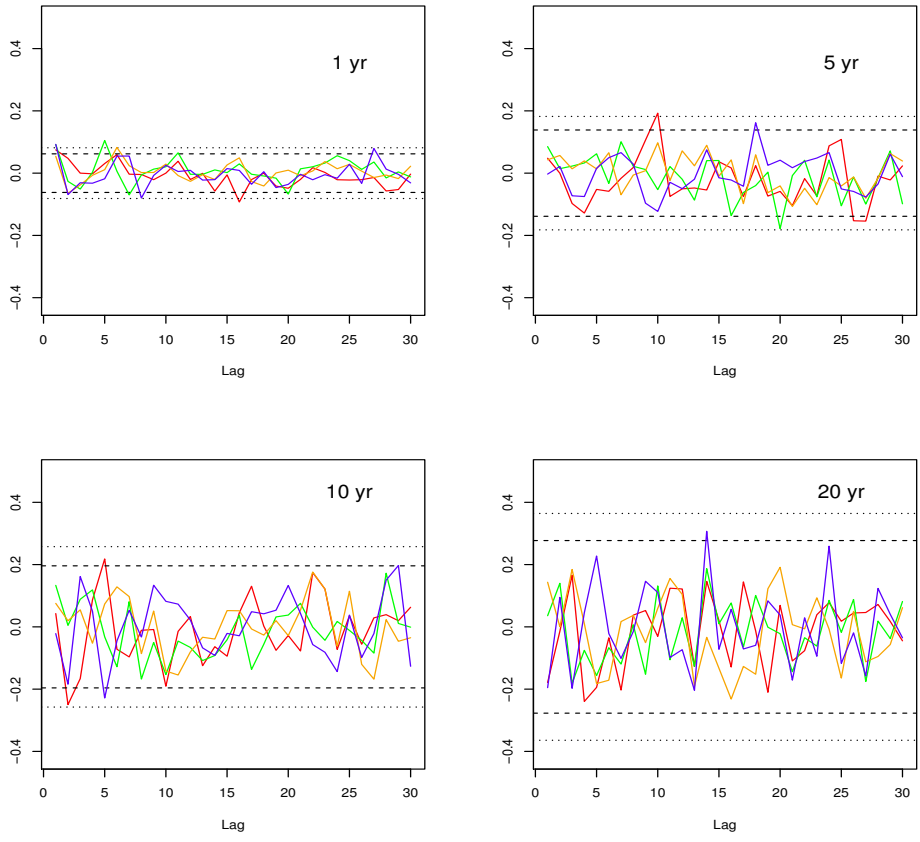

Figure S1.13. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Sol repl}\,i\,t}\} = \{x_{\text{Sol repl.}i\ t} - \bar{x}_{\text{Sol .}t}\}$- sequences, $i = 1, 2, 3, 4$ (North America). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

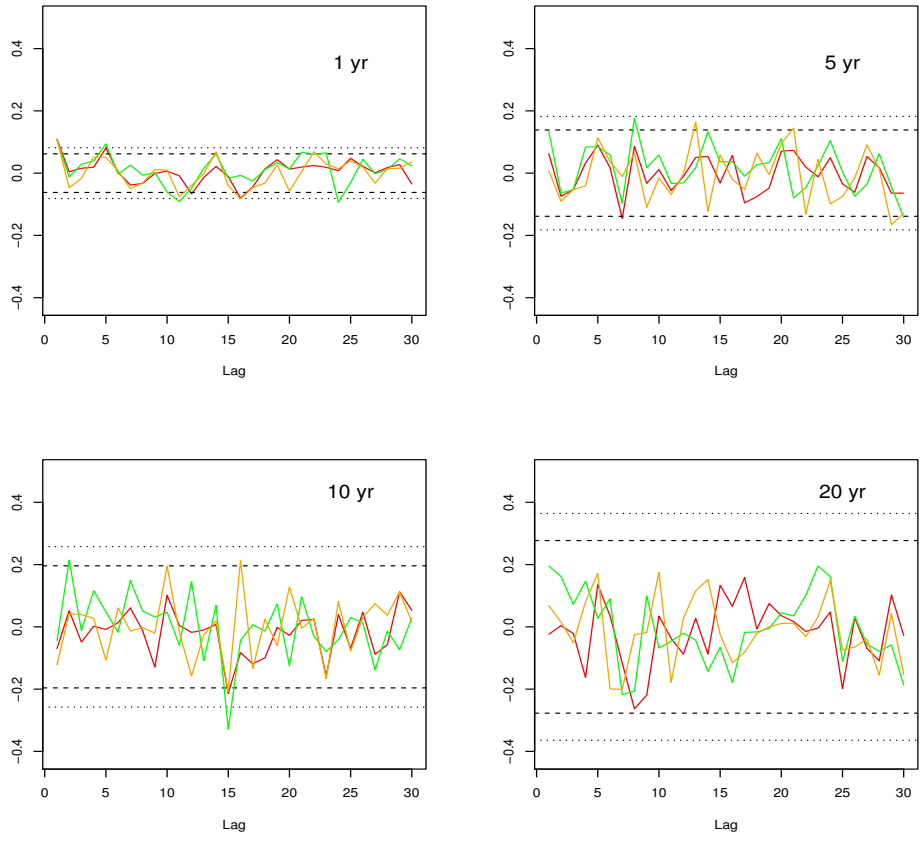

Figure S1.14. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Orb repl}\,i\,t}\} = \{x_{\text{Orb repl.}\,i\,t} - \bar{x}_{\text{Orb}\,.t}\}$- sequences, $i = 1, 2, 3$ (North America). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

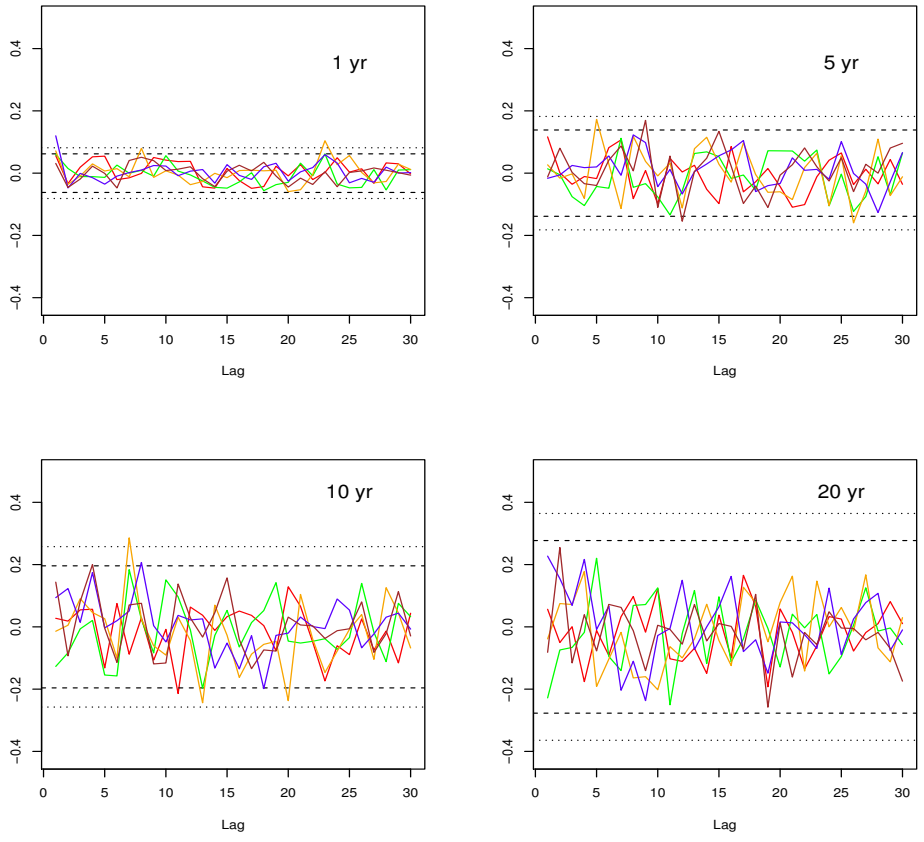

Figure S1.15. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Volc repl}\,i\,t}\} = \{x_{\text{Volc repl.i }t} - \bar{x}_{\text{Volc }.t}\}$- sequences, $i = 1, 2, \ldots, 5$ (North America). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

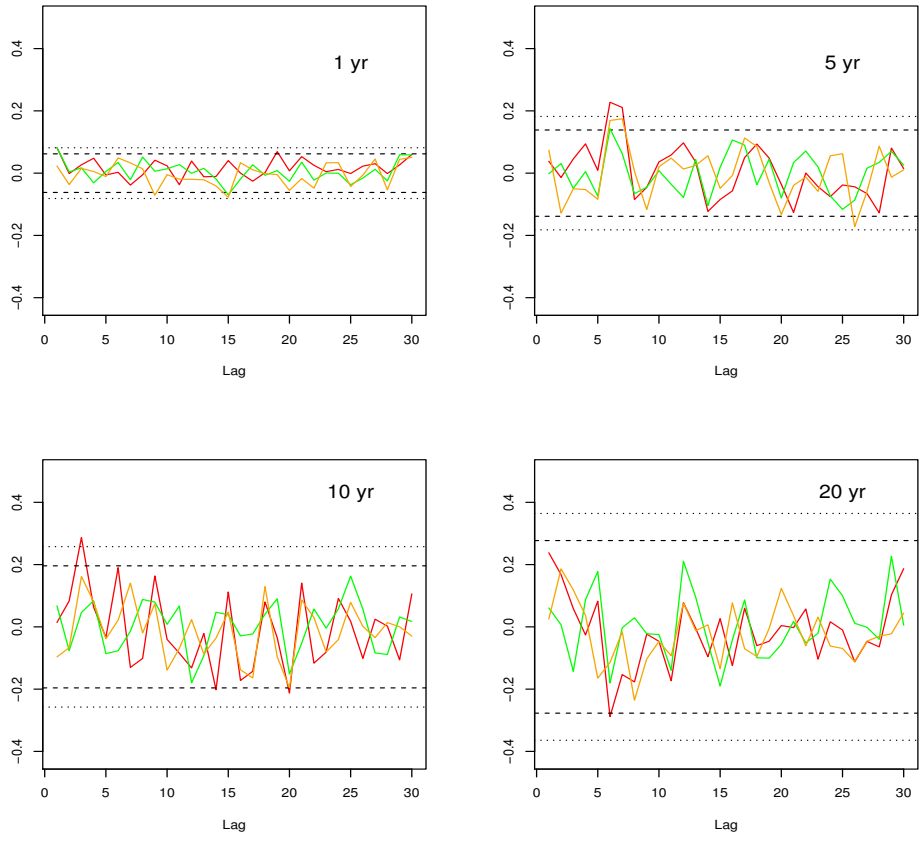

Figure S1.16. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Land repl}\,it}\} = \{x_{\text{Land repl.i}\;t} - \bar{x}_{\text{Land}\;.t}\}$- sequences, $i = 1, 2, 3$ (North America). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

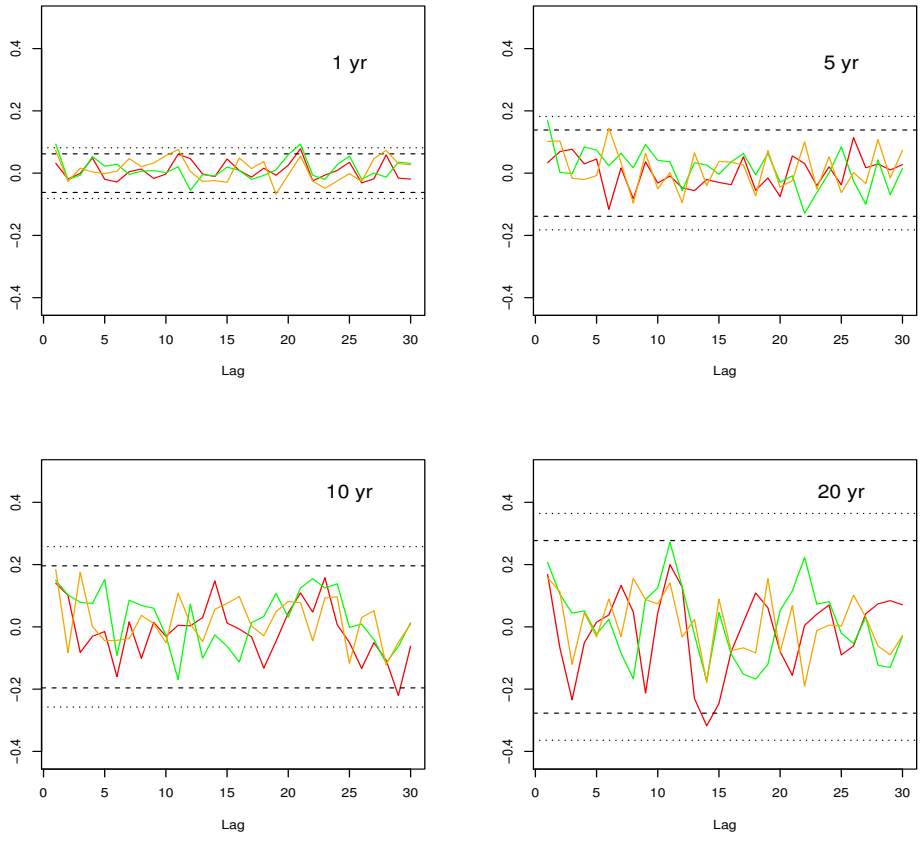

Figure S1.17. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Ghg repl.i }t}\} = \{x_{\text{Ghg repl.i }t} - \bar{x}_{\text{Ghg }.t}\}$- sequences, $i = 1, 2, 3$ (North America). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

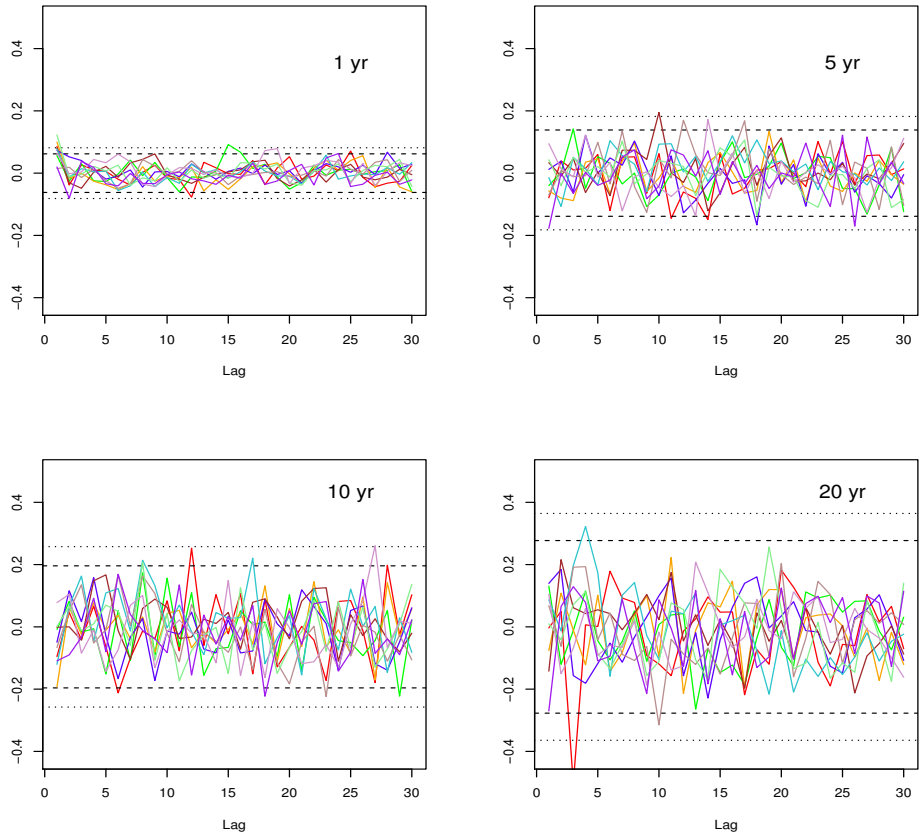

Figure S1.18. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{comb repl}\,it}\} = \{x_{\text{comb repl.i}\;t} - \bar{x}_{\text{comb}\,.t}\}$- sequences, $i = 1, 2, \ldots, 10$ (North America). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

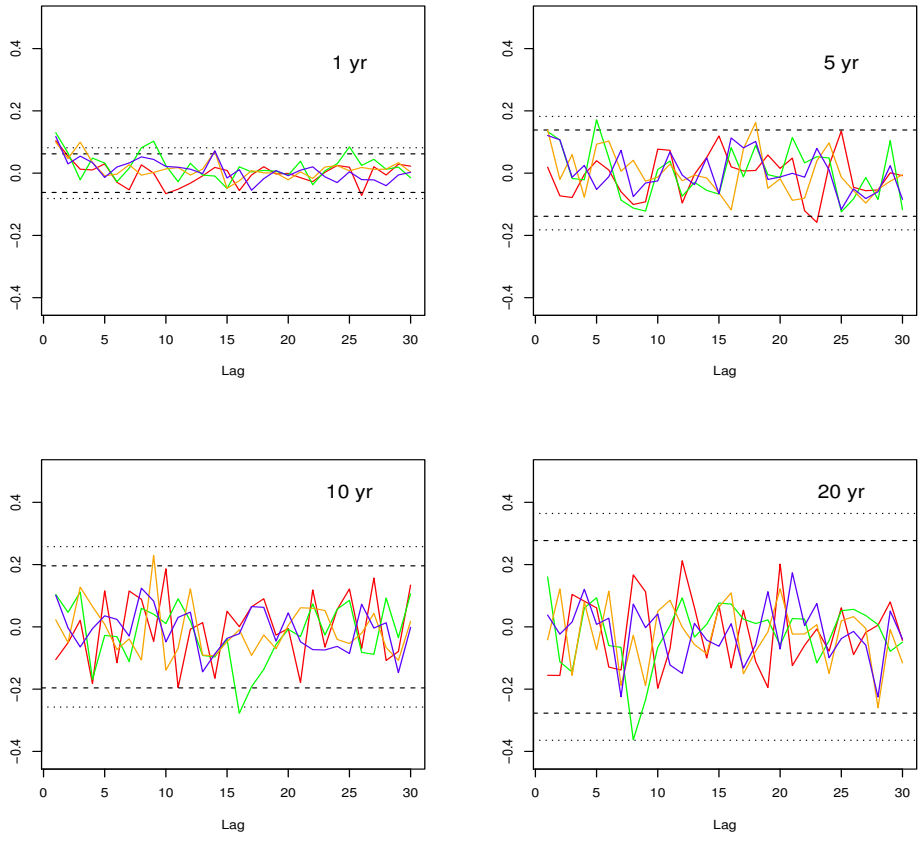

Figure S1.19. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Sol repl}\,i\,t}\} = \{x_{\text{Sol repl.}i\ t} - \bar{x}_{\text{Sol .}t}\}$- sequences, $i = 1, 2, 3, 4$ (Asia). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

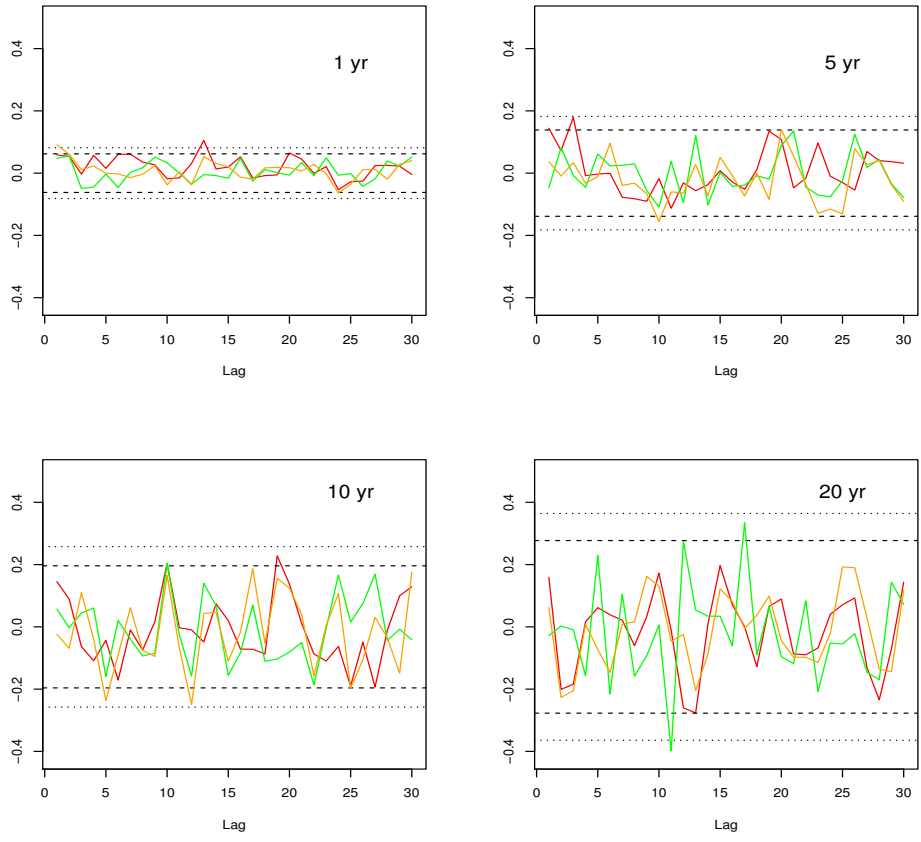

Figure S1.20. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Orb repl}\,i\,t}\} = \{x_{\text{Orb repl.i }t} - \bar{x}_{\text{Orb .}t}\}$- sequences, $i = 1, 2, 3$ (Asia). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

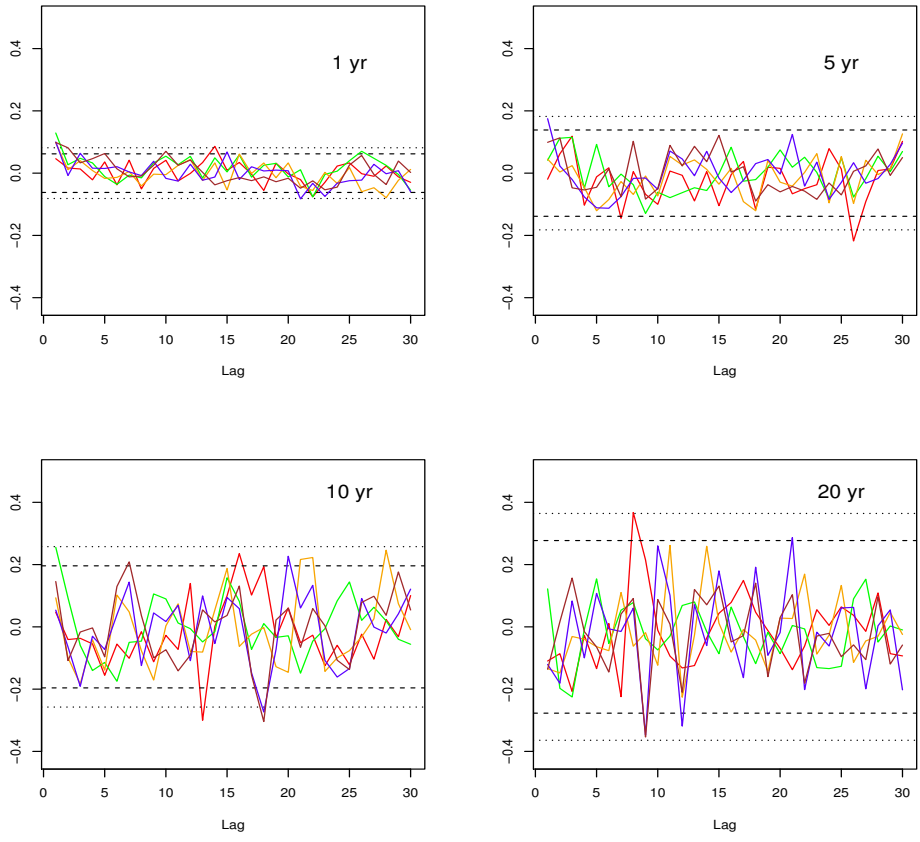

Figure S1.21. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Volc repl}\,i\,t}\} = \{x_{\text{Volc repl.i }t} - \bar{x}_{\text{Volc .}t}\}$- sequences, $i = 1, 2, \ldots, 5$ (Asia). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

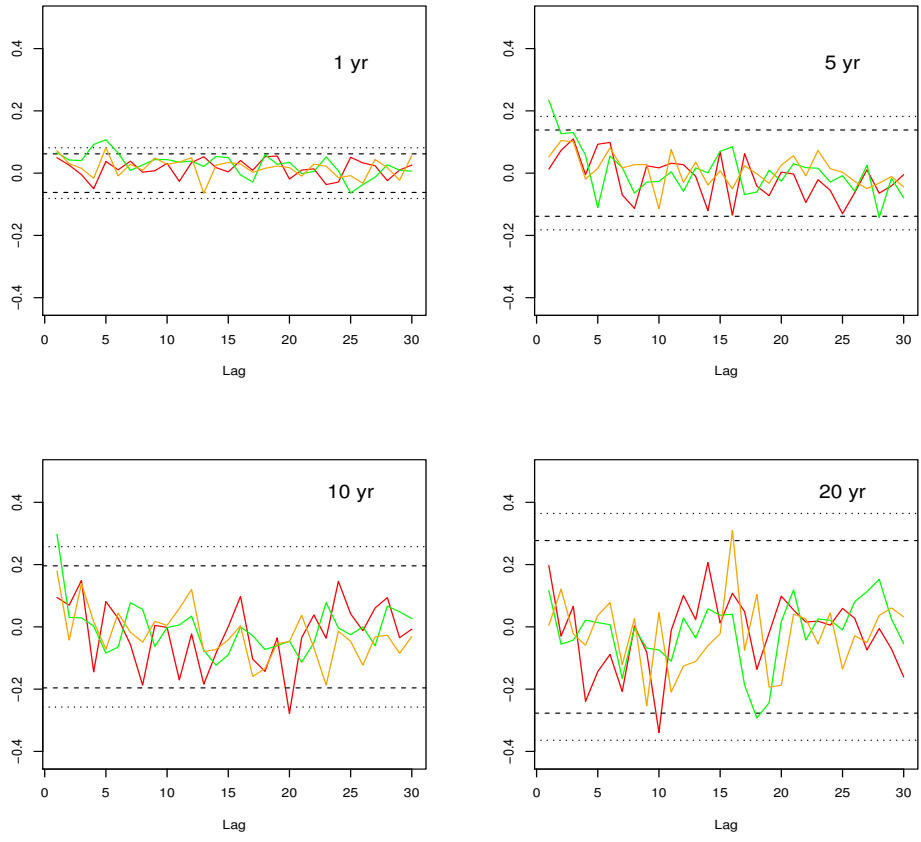

Figure S1.22. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Land repl}\,it}\} = \{x_{\text{Land repl.i}\,t} - \bar{x}_{\text{Land}\,.t}\}$- sequences, $i = 1, 2, 3$ (Asia). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

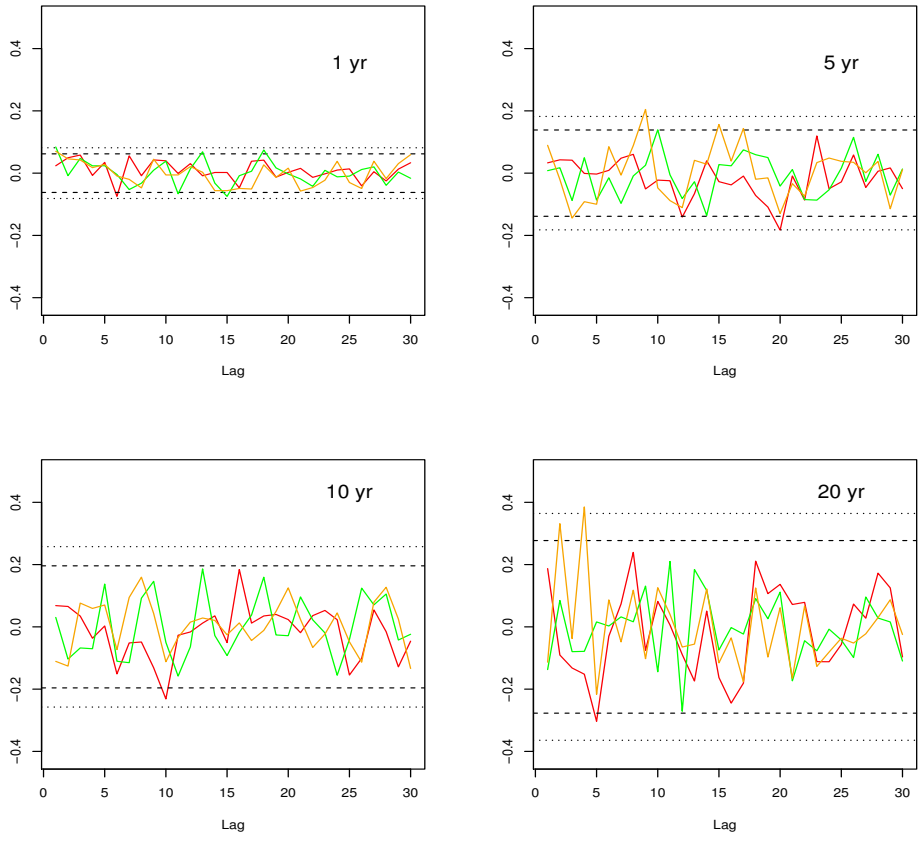

Figure S1.23. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Ghg repl.i}\,t}\} = \{x_{\text{Ghg repl.i}\,t} - \bar{x}_{\text{Ghg}\,.t}\}$- sequences, $i = 1, 2, 3$ (Asia). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

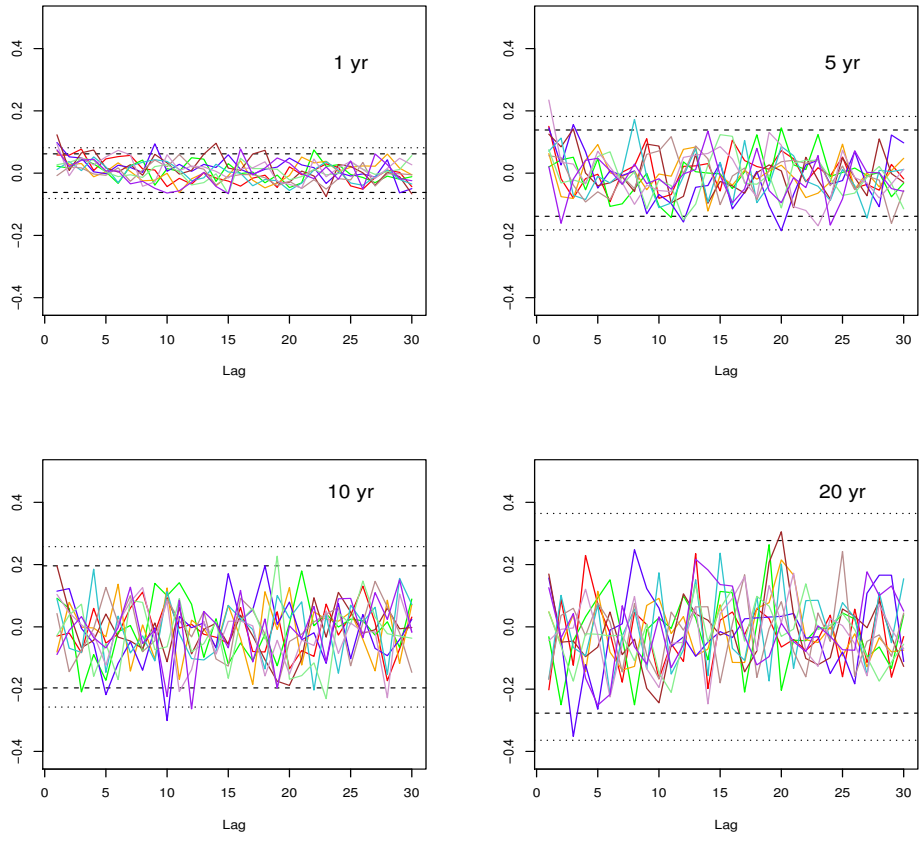

Figure S1.24. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{comb repl}\,it}\} = \{x_{\text{comb repl.i}\;t} - \bar{x}_{\text{comb}\;.t}\}$- sequences, $i = 1, 2, \ldots, 10$ (Asia). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

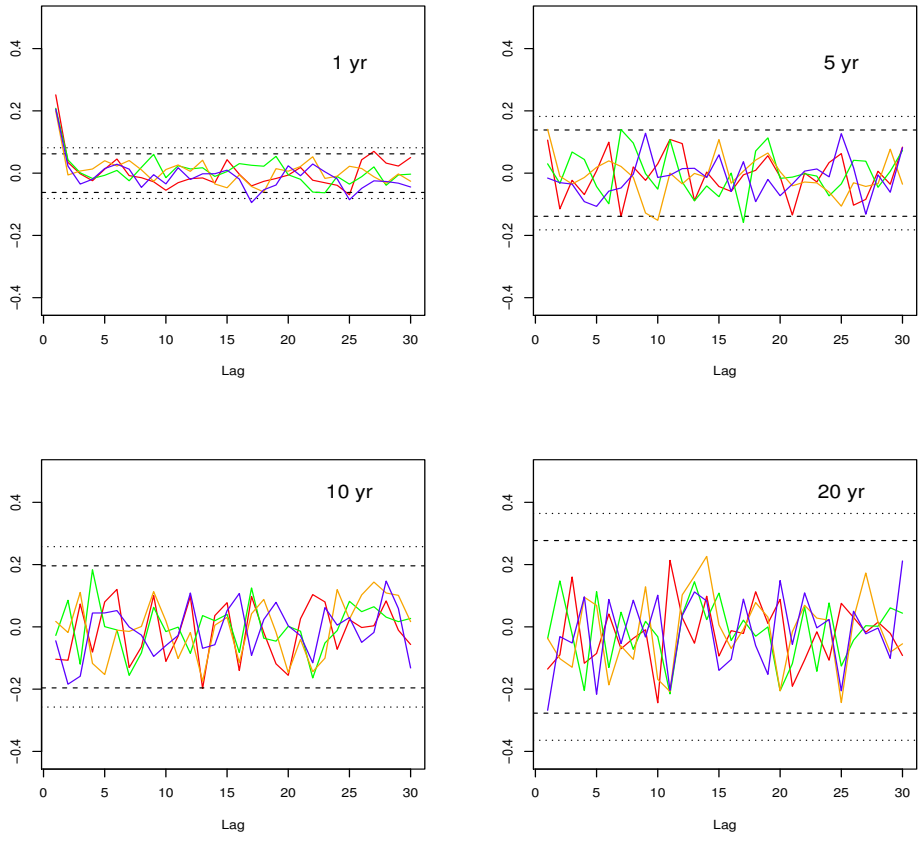

Figure S1.25. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Sol repl}\,i\,t}\} = \{x_{\text{Sol repl.}i\ t} - \bar{x}_{\text{Sol .}t}\}$- sequences, $i = 1, 2, 3, 4$ (South America). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

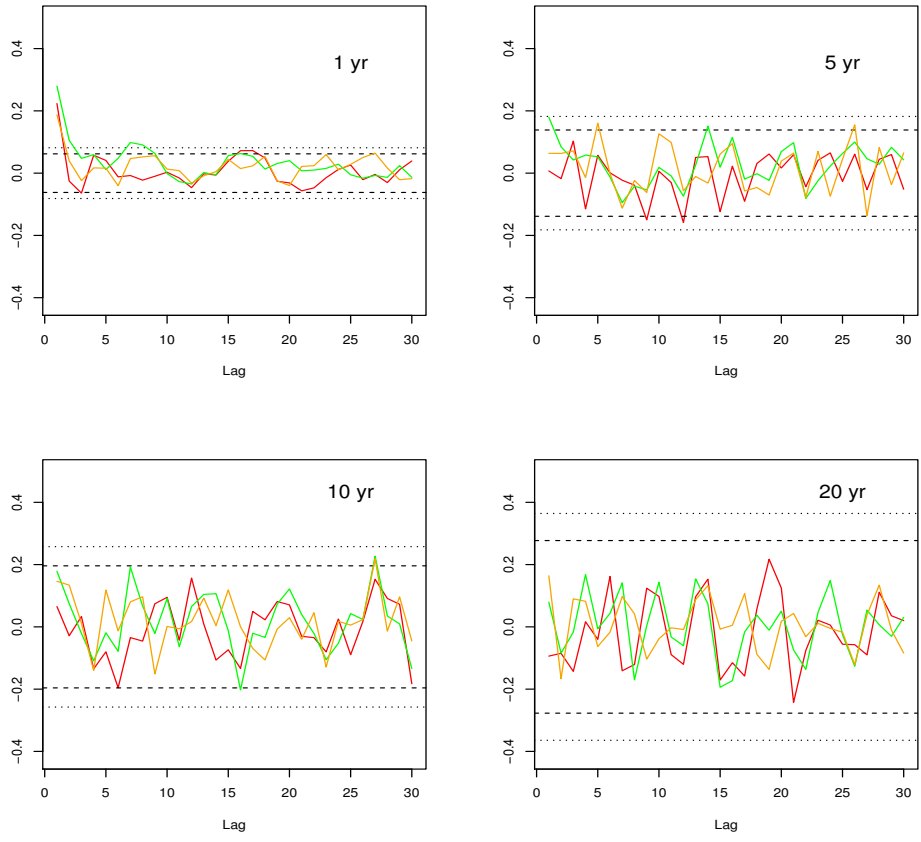

Figure S1.26. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Orb repl}\,i\,t}\} = \{x_{\text{Orb repl.}i\ t} - \bar{x}_{\text{Orb .}t}\}$- sequences, $i = 1, 2, 3$ (South America). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

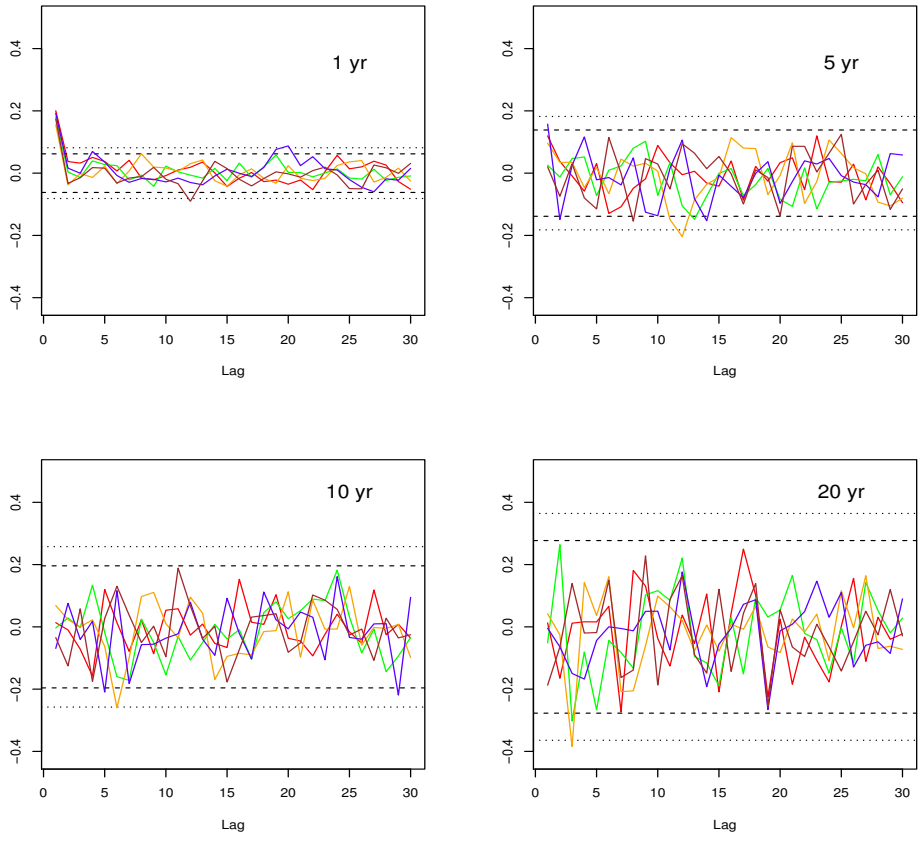

Figure S1.27. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Volc repl}\,i\,t}\} = \{x_{\text{Volc repl.i}\ t} - \bar{x}_{\text{Volc}\ .t}\}$- sequences, $i = 1, 2, \ldots, 5$ (South America). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

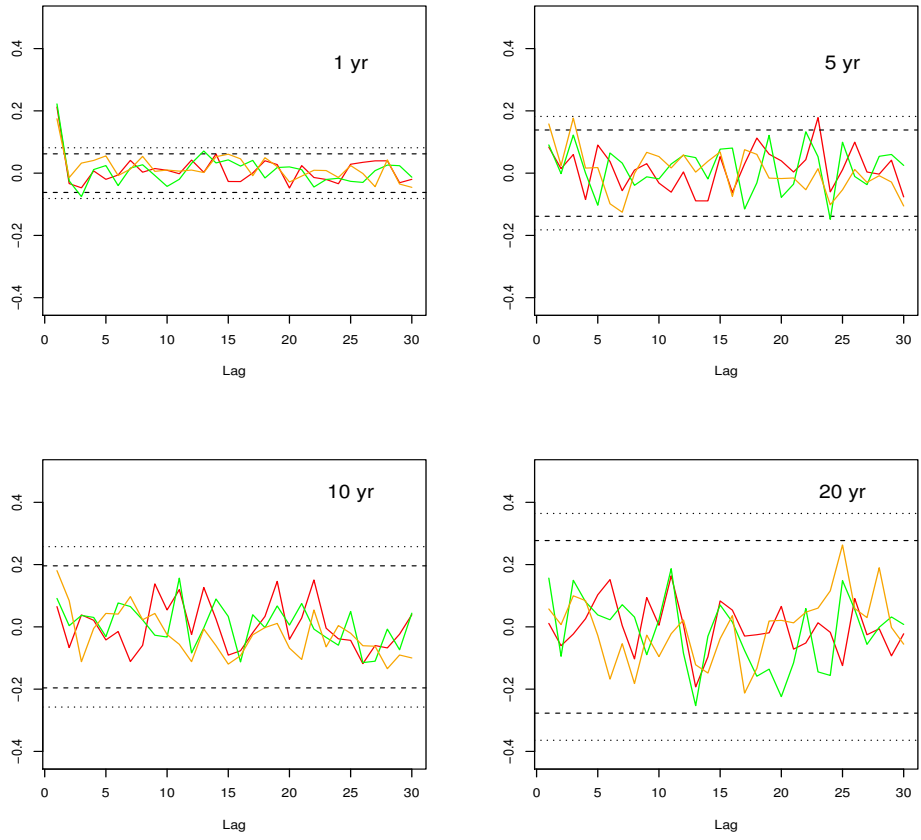

Figure S1.28. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Land repl}\,it}\} = \{x_{\text{Land repl.i}\;t} - \bar{x}_{\text{Land}\,.t}\}$- sequences, $i = 1, 2, 3$ (South America). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

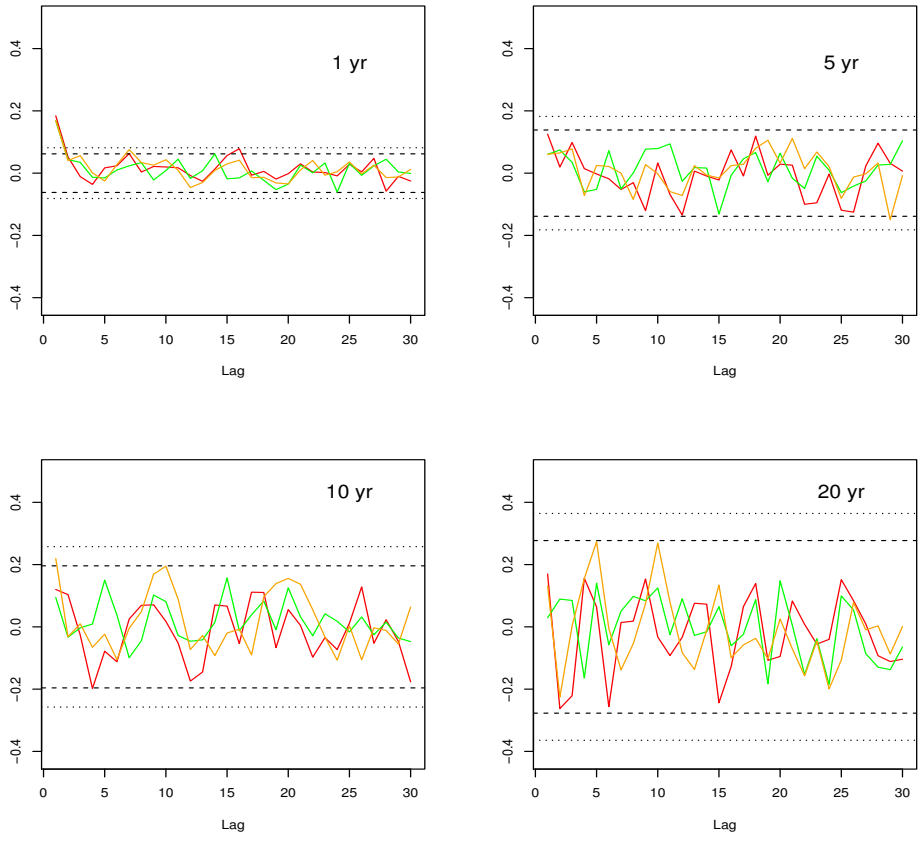

Figure S1.29. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Ghg repl.i}\,t}\} = \{x_{\text{Ghg repl.i}\,t} - \bar{x}_{\text{Ghg}\,.t}\}$- sequences, $i = 1, 2, 3$ (South America). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

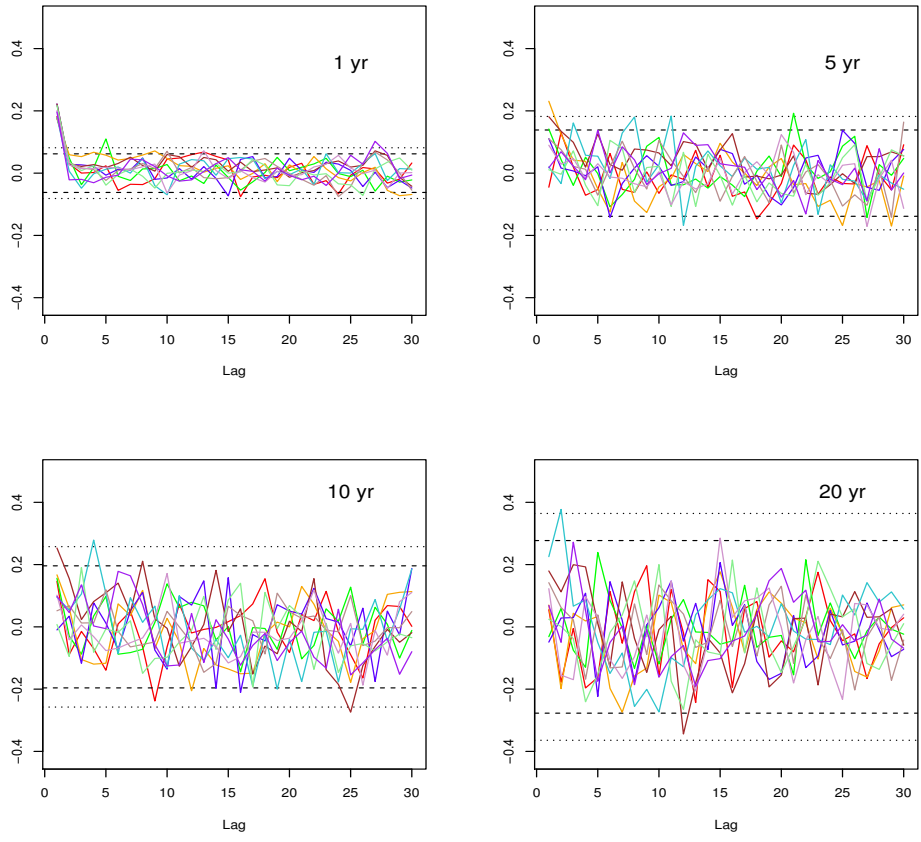

Figure S1.30. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{comb repl}\,it}\} = \{x_{\text{comb repl.i}\;t} - \bar{x}_{\text{comb}\,.t}\}$- sequences, $i = 1, 2, \ldots, 10$ (South America). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

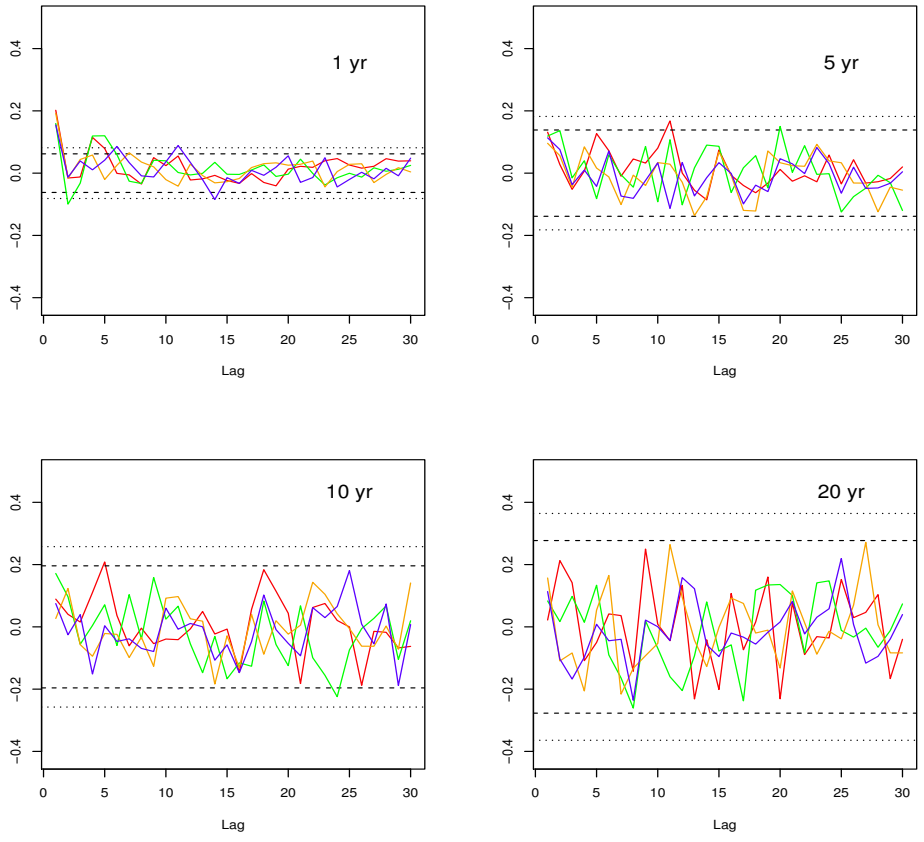

Figure S1.31. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Sol repl i}\,t}\} = \{x_{\text{Sol repl.i}\,t} - \bar{x}_{\text{Sol .}t}\}$- sequences, $i = 1, 2, 3, 4$ (Australasia). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

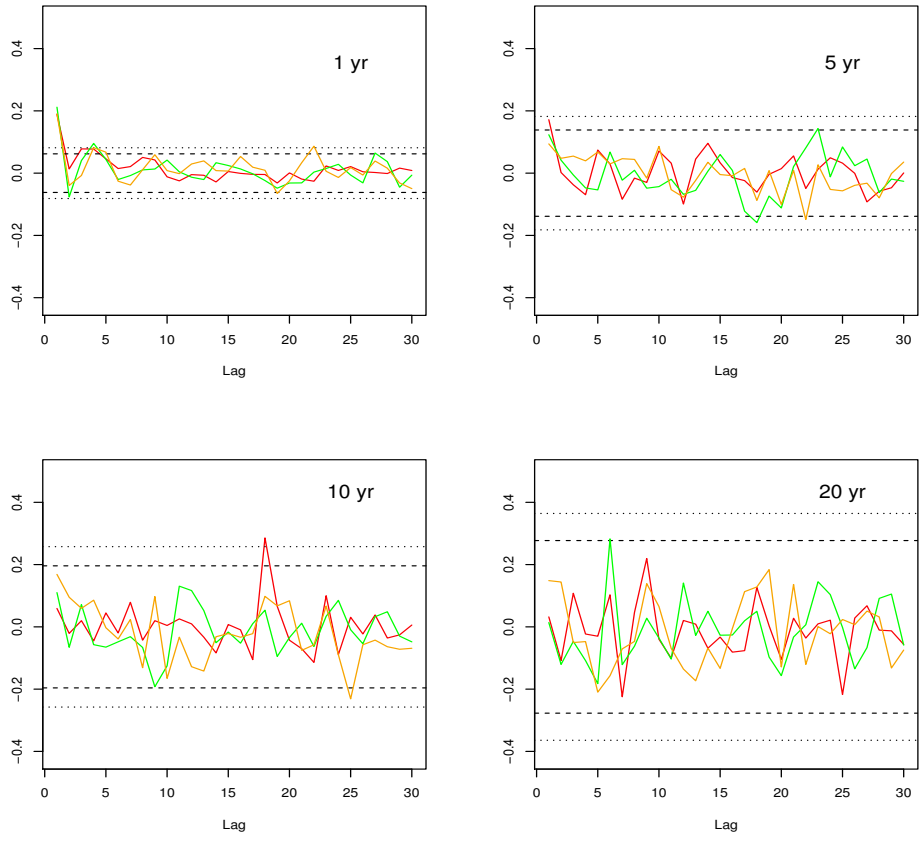

Figure S1.32. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Orb repl}\,i\,t}\} = \{x_{\text{Orb repl.}i\ t} - \bar{x}_{\text{Orb}\,.t}\}$- sequences, $i = 1, 2, 3$ (Australasia). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

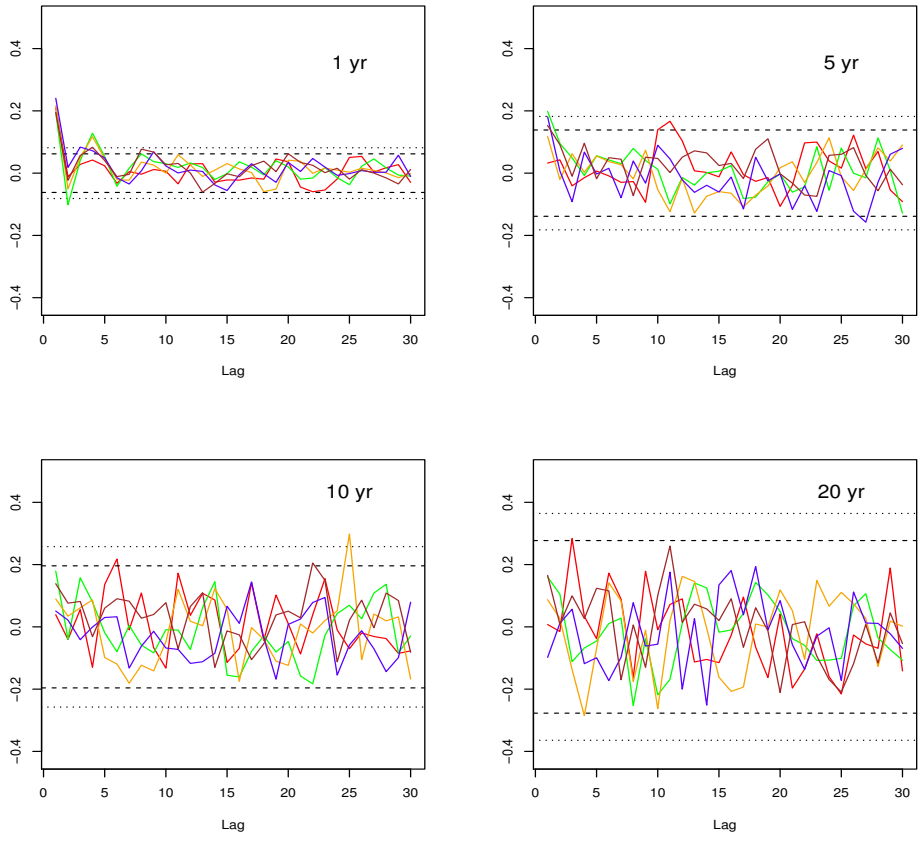

Figure S1.33. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Volc repl}\,i\,t}\} = \{x_{\text{Volc repl.i}\ t} - \bar{x}_{\text{Volc}\ .t}\}$- sequences, $i = 1, 2, \ldots, 5$ (Australasia). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

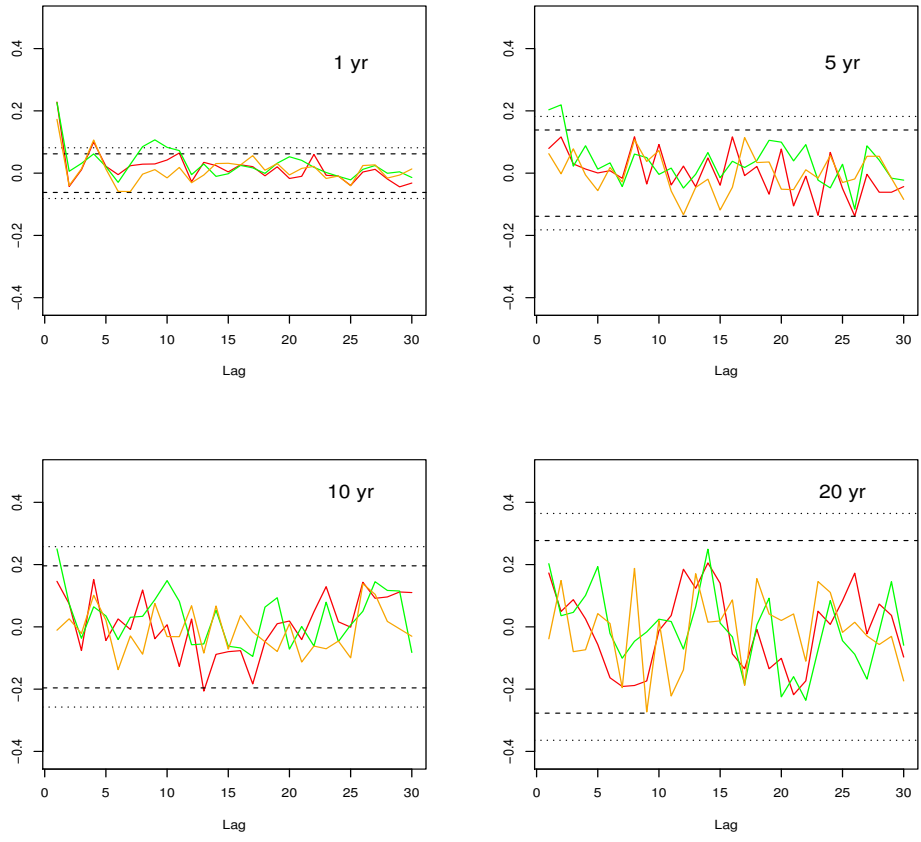

Figure S1.34. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Land repl}\,it}\} = \{x_{\text{Land repl.i}\;t} - \bar{x}_{\text{Land}\;.t}\}$- sequences, $i = 1, 2, 3$ (Australasia). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

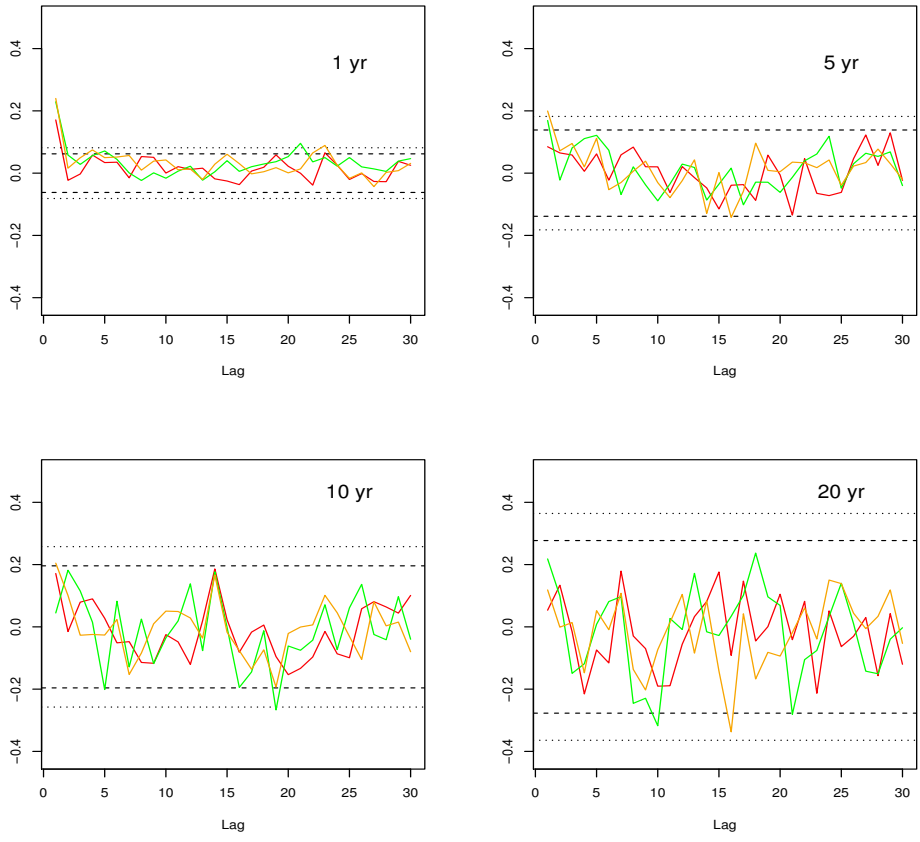

Figure S1.35. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Ghg repl.i}\,t}\} = \{x_{\text{Ghg repl.i}\,t} - \bar{x}_{\text{Ghg}\,.t}\}$- sequences, $i = 1, 2, 3$ (Australasia). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

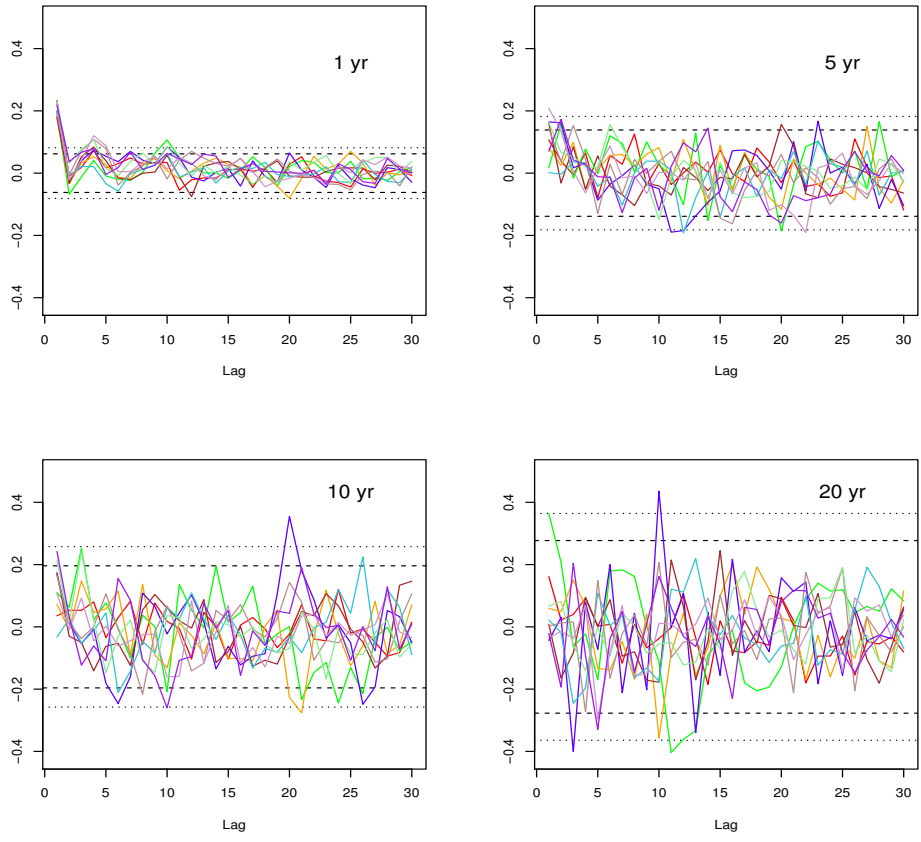

Figure S1.36. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{comb repl}\,it}\} = \{x_{\text{comb repl.i}\;t} - \bar{x}_{\text{comb}\;.t}\}$- sequences, $i = 1, 2, \ldots, 10$ (Australasia). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

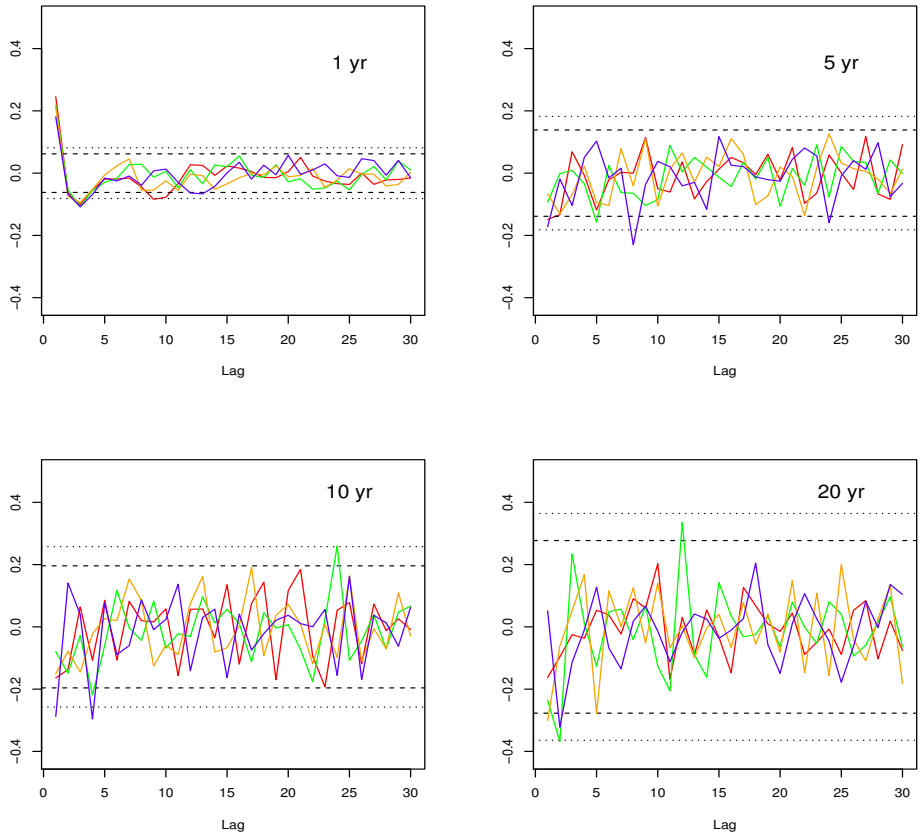

Figure S1.37. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Sol repl}\,i\,t}\} = \{x_{\text{Sol repl.}i\ t} - \bar{x}_{\text{Sol .}t}\}$- sequences, $i = 1, 2, 3, 4$ (Antarctica). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

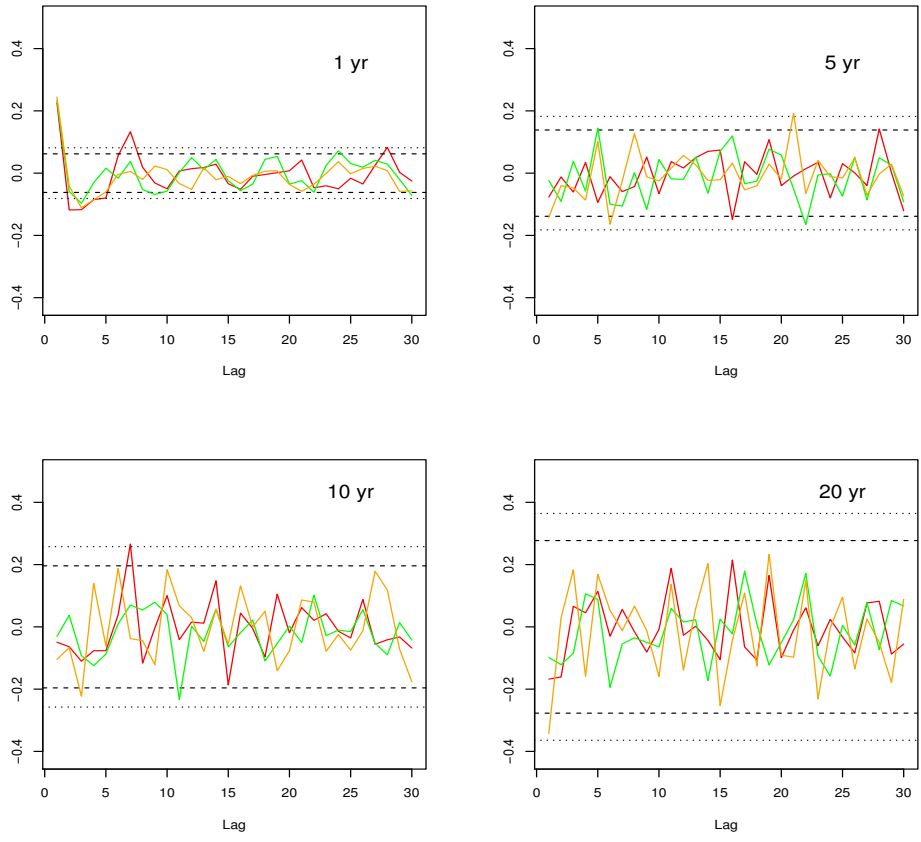

Figure S1.38. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Orb repl}\,i\,t}\} = \{x_{\text{Orb repl.}i\ t} - \bar{x}_{\text{Orb}\,.t}\}$- sequences, $i = 1, 2, 3$ (Antarctica). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

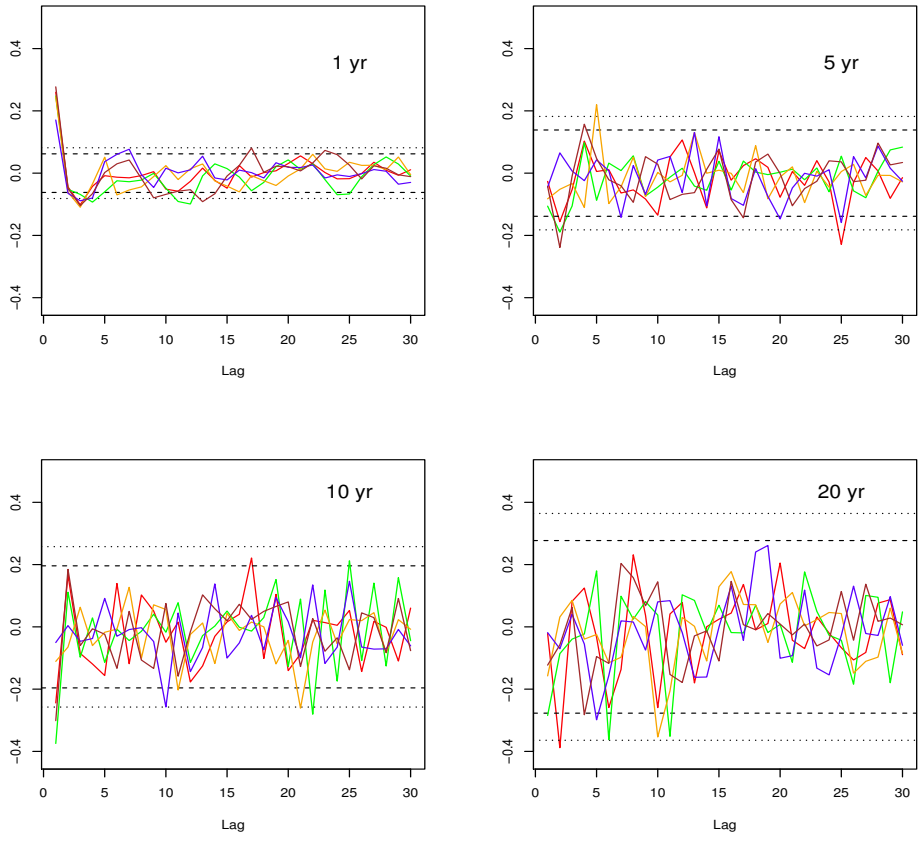

Figure S1.39. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Volc repl}\,i\,t}\} = \{x_{\text{Volc repl.i}\,t} - \bar{x}_{\text{Volc}\,.t}\}$- sequences, $i = 1, 2, \ldots, 5$ (Antarctica). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

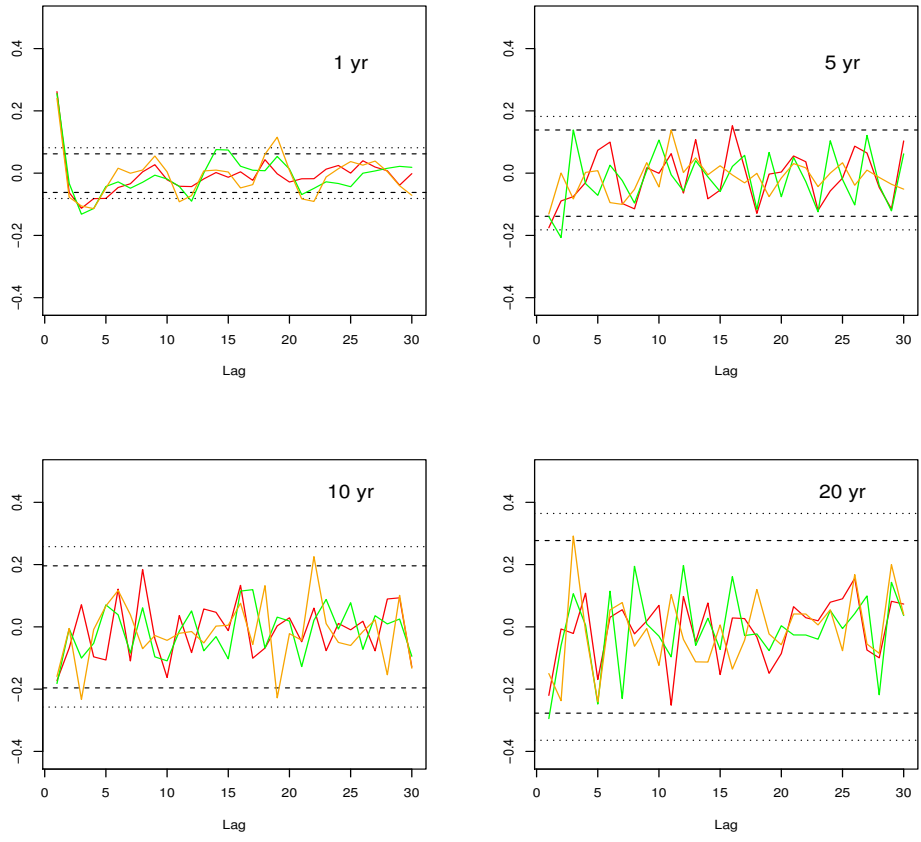

Figure S1.40. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{Land repl}\,it}\} = \{x_{\text{Land repl.i}\;t} - \bar{x}_{\text{Land}\;.t}\}$- sequences, $i = 1, 2, 3$ (Antarctica). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

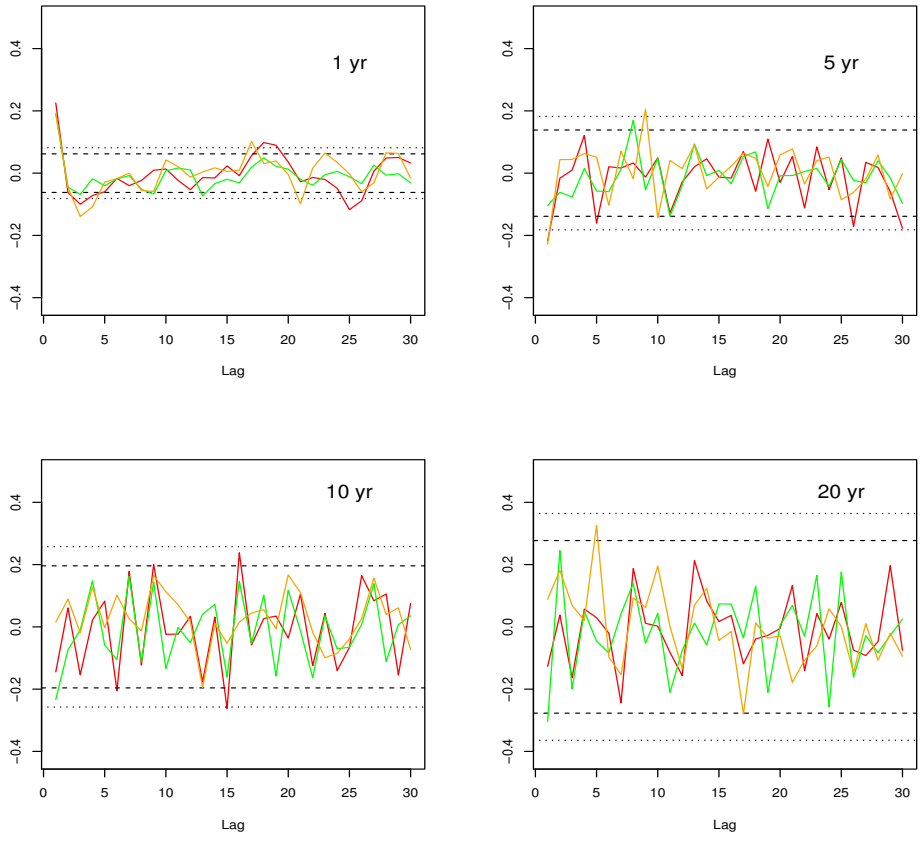

Figure S1.41. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\mathsf{Ghg\ repl.}i\,t}\} = \{x_{\mathsf{Ghg\ repl.}i\,t} - \bar{x}_{\mathsf{Ghg}\,.t}\}$- sequences, $i = 1, 2, 3$ (Antarctica). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

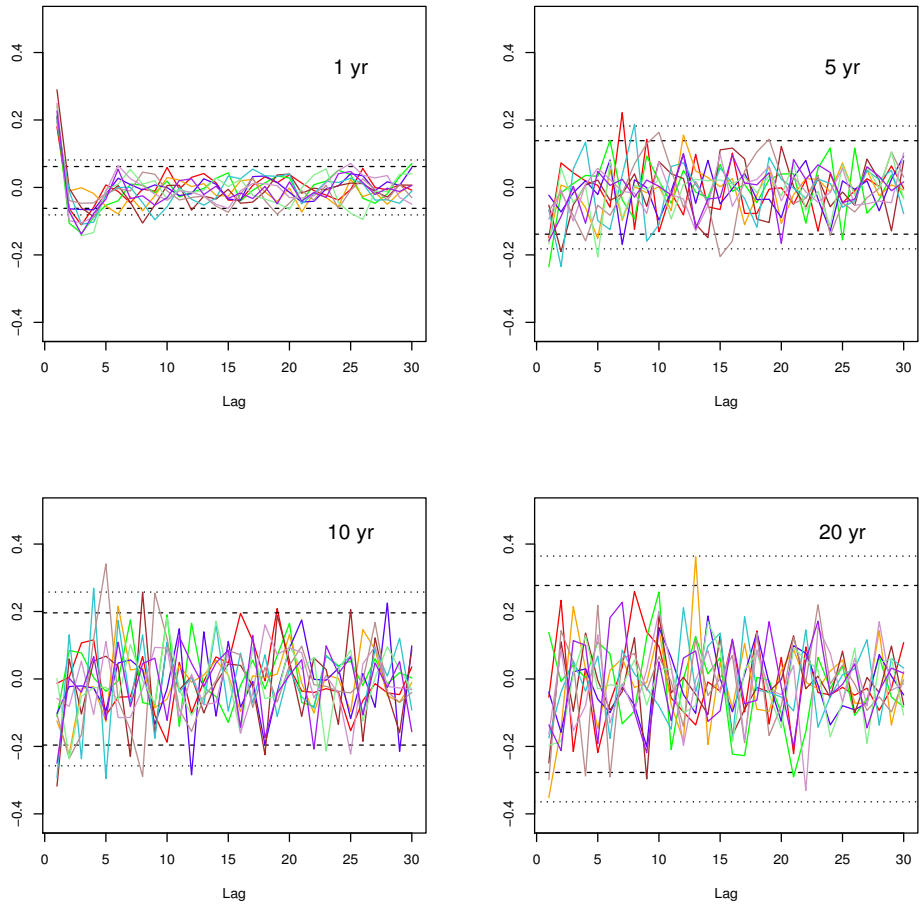

Figure S1.42. The sample ACF function up to 30 lags for the residual $\{\tilde{\delta}_{\text{comb repl}\,it}\} = \{x_{\text{comb repl.i}\;t} - \bar{x}_{\text{comb}\;.t}\}$- sequences, $i = 1, 2, \ldots, 10$ (Antarctica). The two-sided 95% and 99% bounds, denoted by dashed lines, are equal to $\pm 1.96/\sqrt{n}$ and $\pm 2.58/\sqrt{n}$, respectively, where $n = 1000/m$, where $m = 1$, 5, 10, 20.

2 Plots of decadally resolved time series

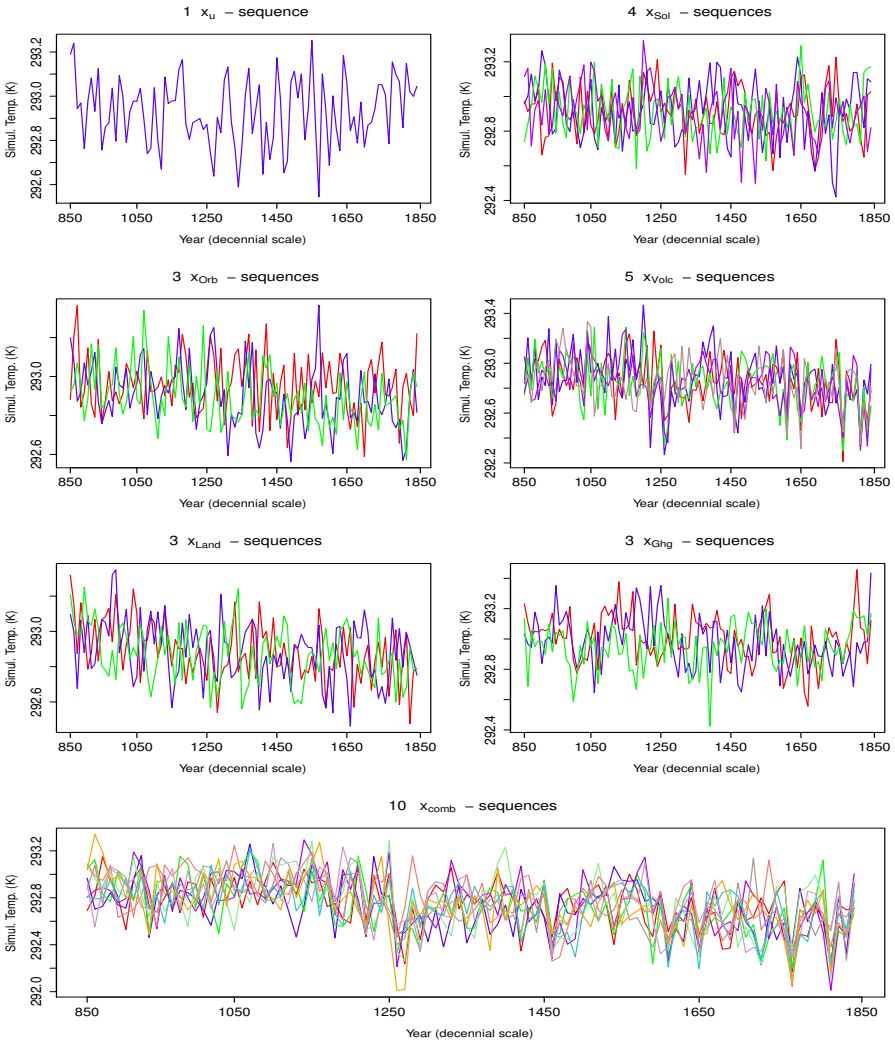

Figure S2.1. Plots of the *Europe* data resolved at a decennial scale.

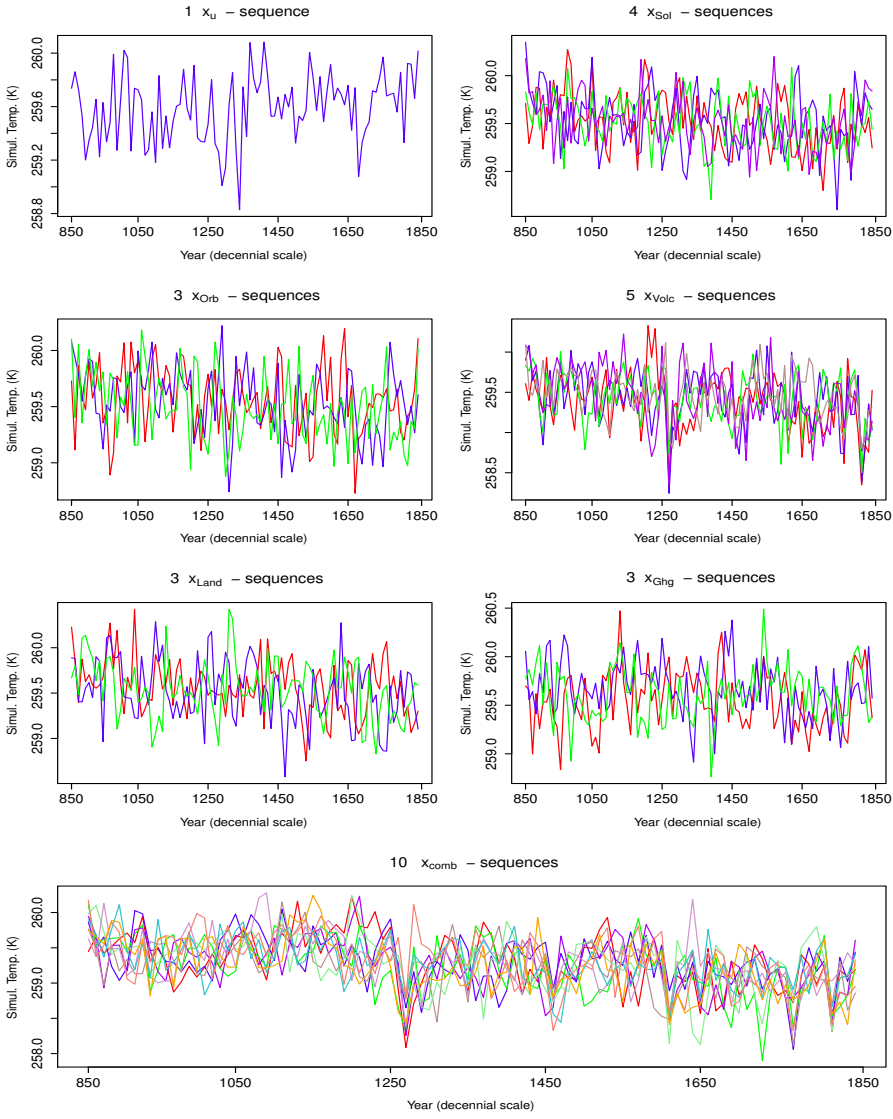

Figure S2.2. Plots of the *Arctic* data resolved at a decennial scale.

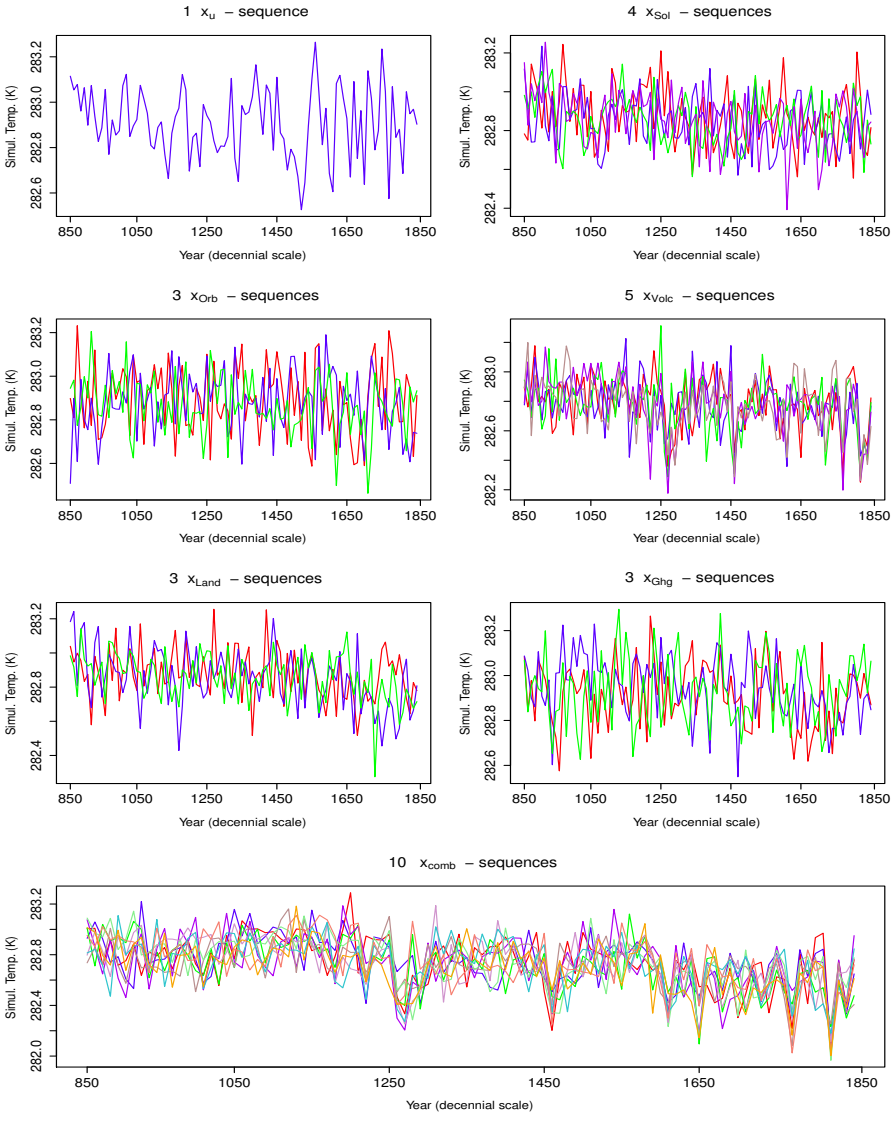

Figure S2.3. Plots of the *North America* data resolved at a decennial scale.

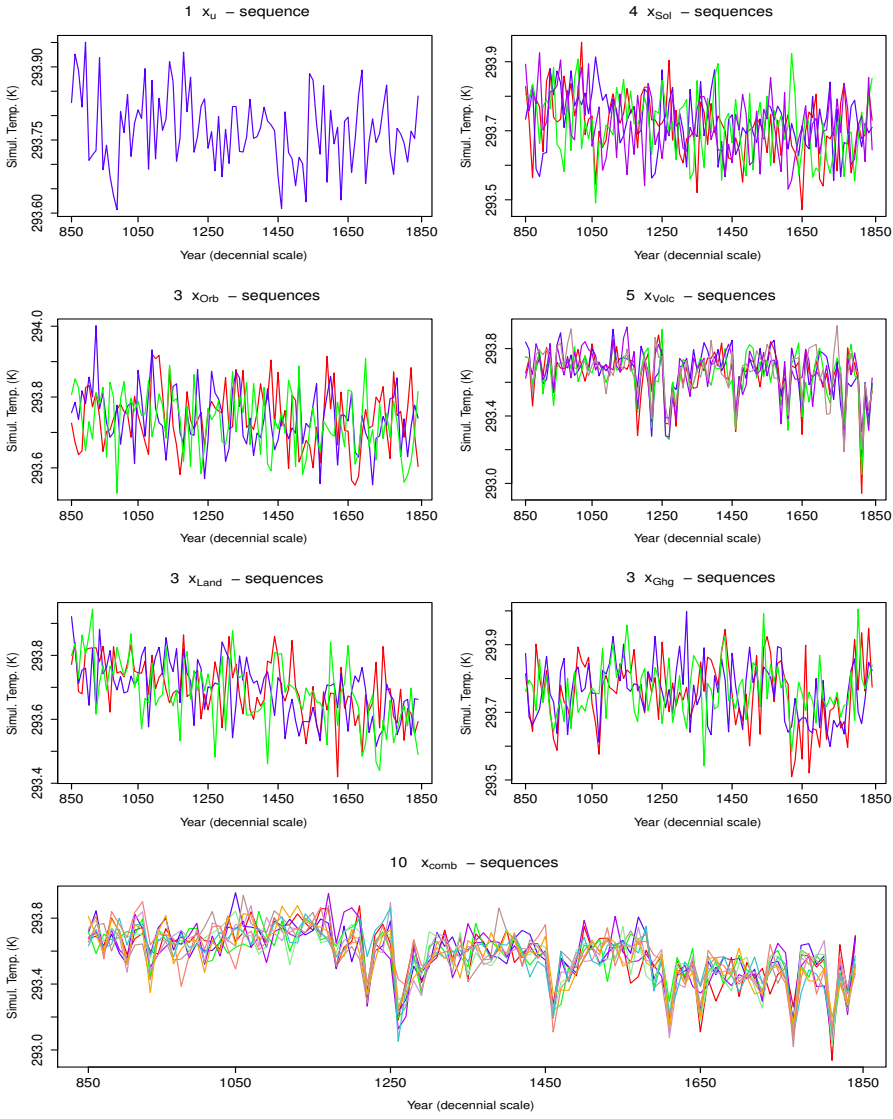

Figure S2.4. Plots of the *Asia* data resolved at a decennial scale.

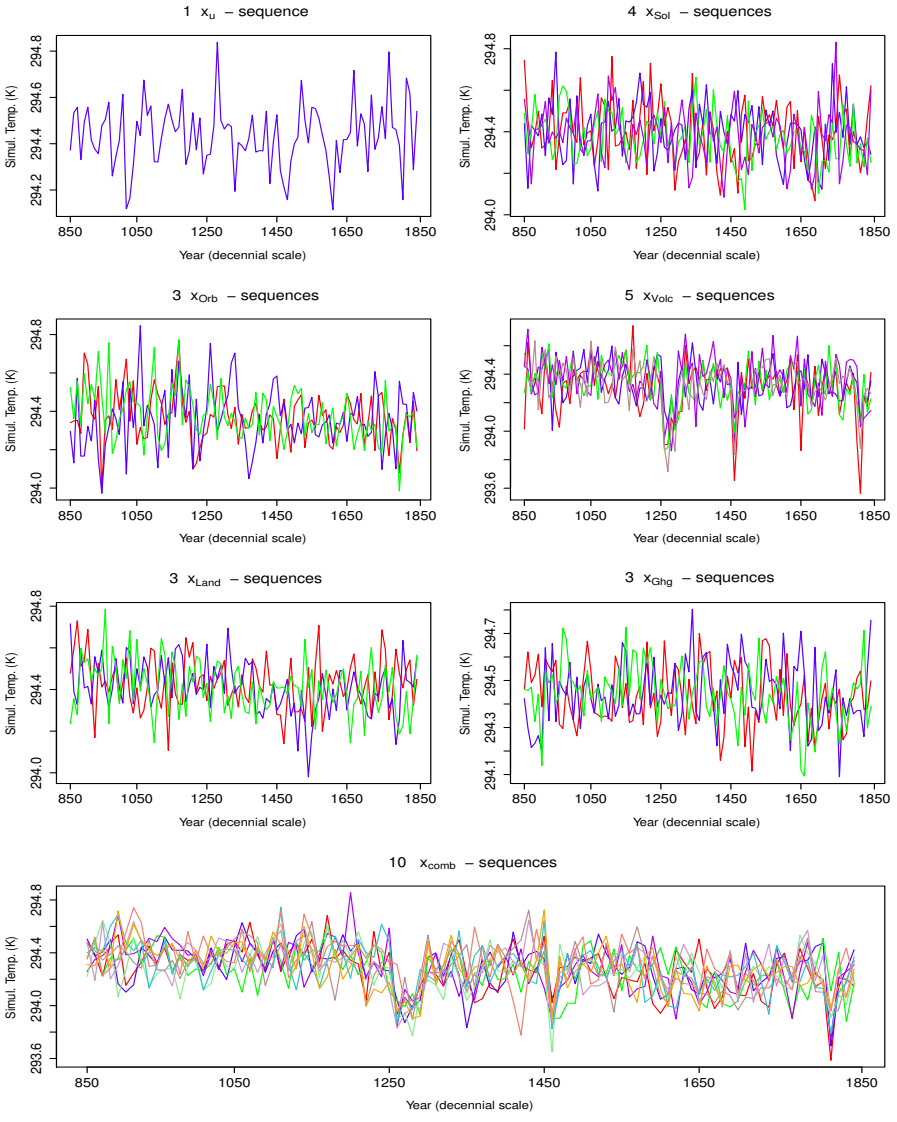

Figure S2.5. Plots of the *South America* data resolved at a decennial scale.

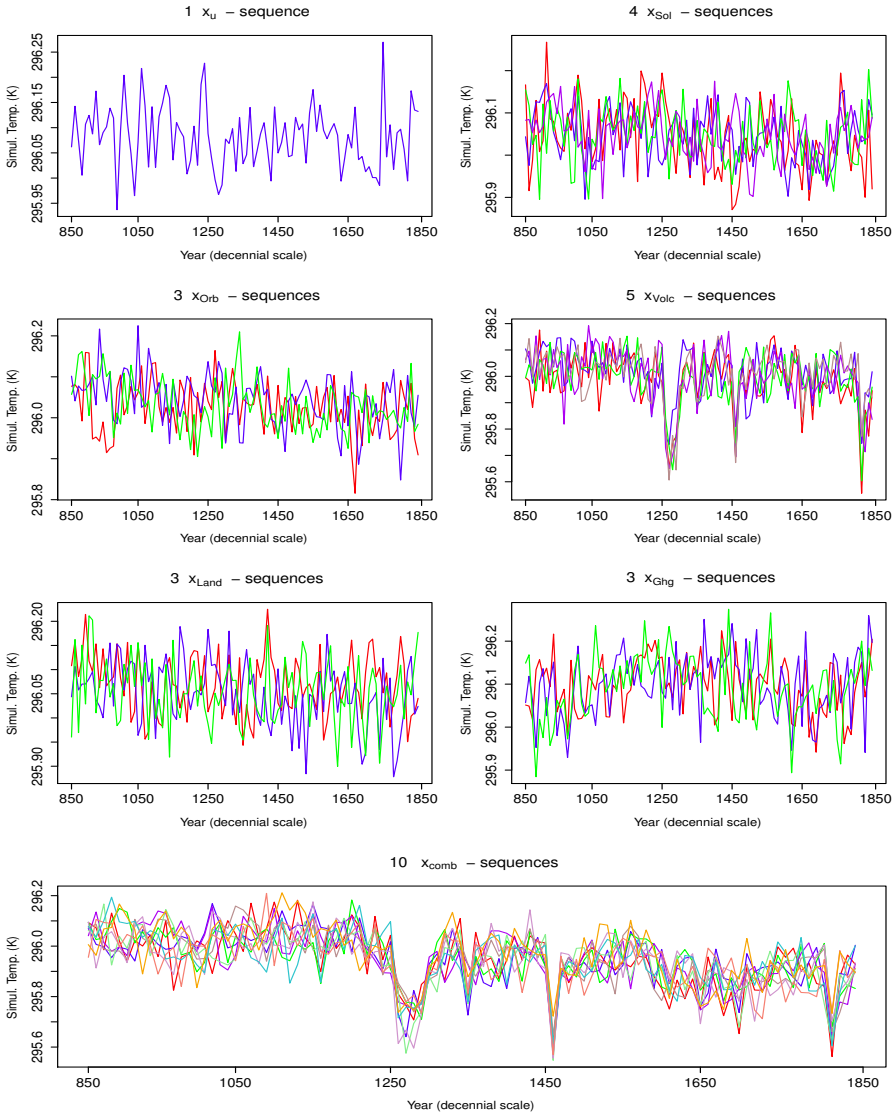

Figure S2.6. Plots of the *Australasia* data resolved at a decennial scale.

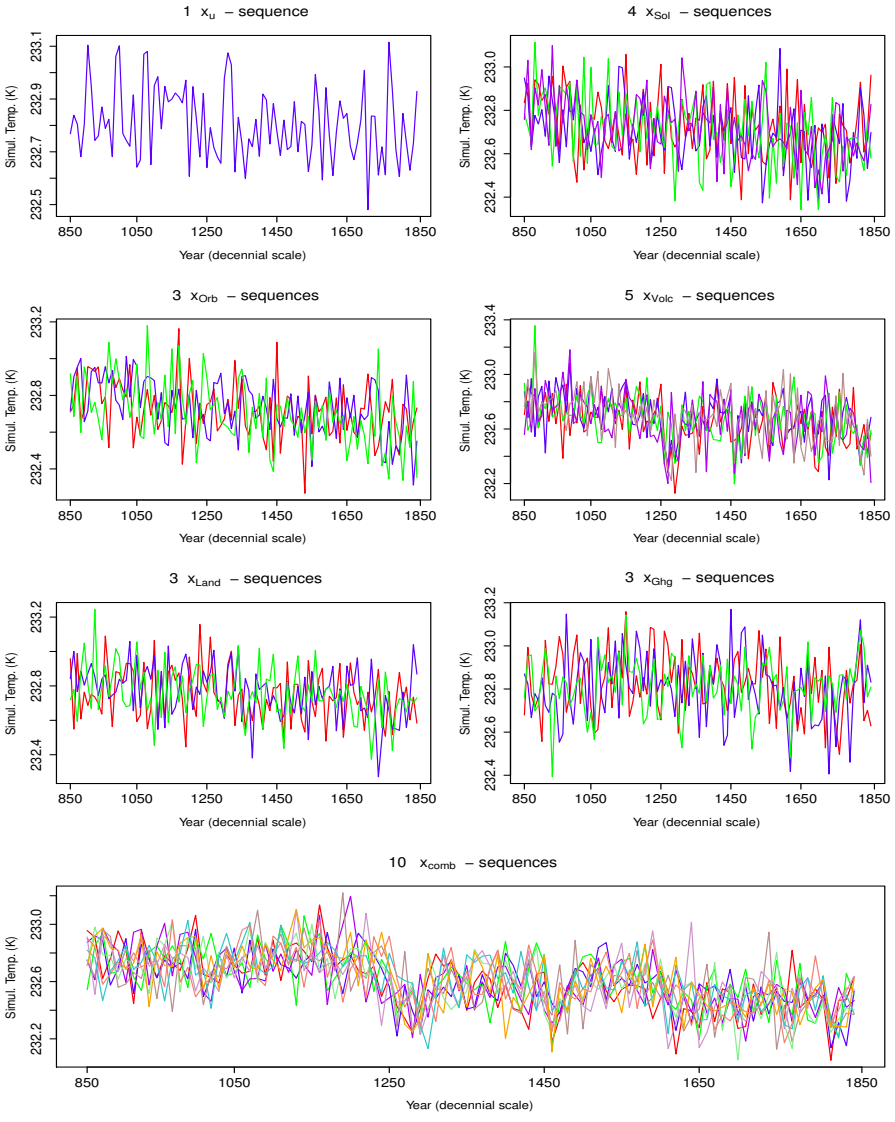

Figure S2.7. Plots of the *Antarctica* data resolved at a decennial scale.

3 Estimated densities of the decadally resolved residual $\tilde{\delta}$ - sequences

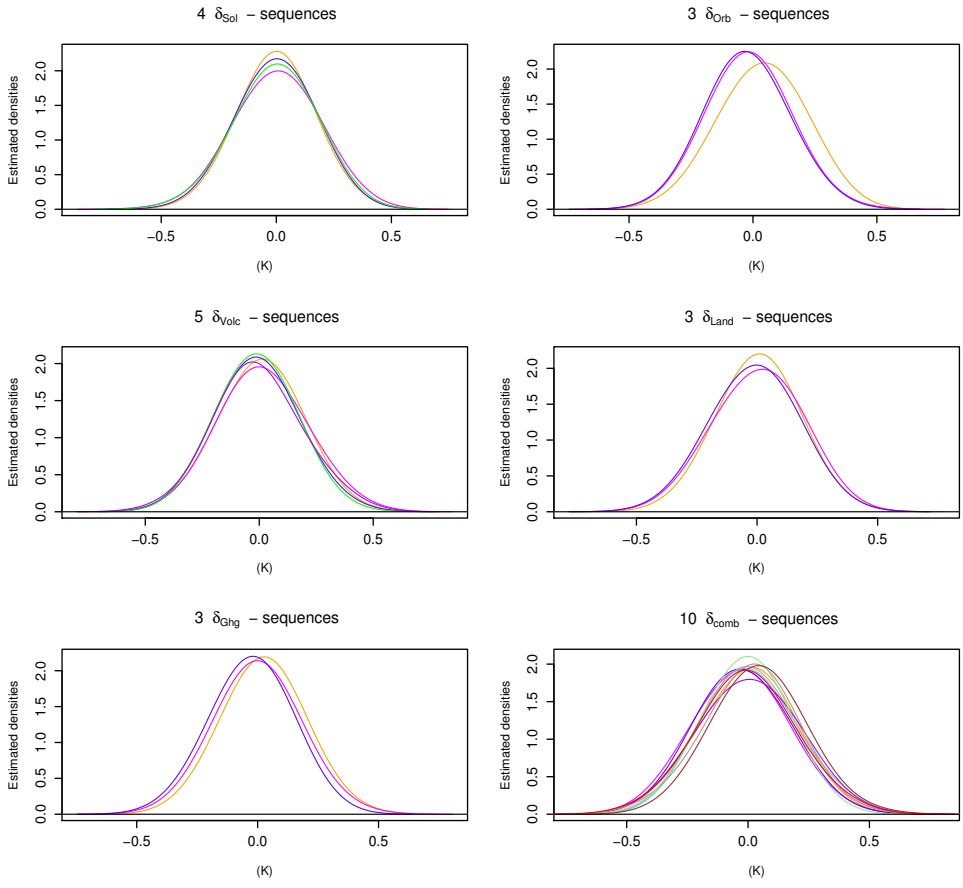

Figure S3.1. Estimated densities for the $\{\tilde{\delta}_{\text{forcing}\,t}\} = \{x_{\text{forcing, repl. i}\,t} - \bar{x}_{\text{forcing .}\ t}\}$-sequences, where forcing is solar, orbital, volcanic, land use/land cover, well-mixed greenhouse gases, and the combination of all forcings (Europe). All time series have the time unit of 10 years.

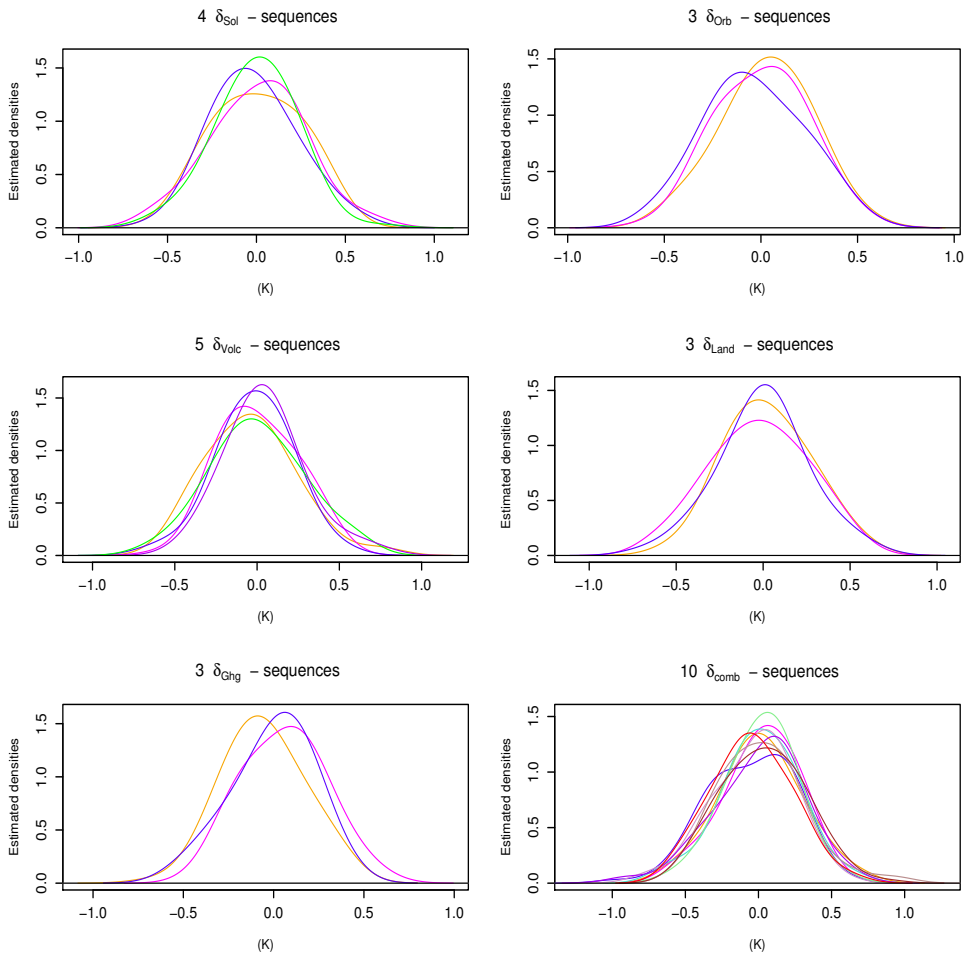

Figure S3.2. Estimated densities for the $\{\tilde{\delta}_{\text{forcing}\,t}\} = \{x_{\text{forcing, repl. i}\,t} - \bar{x}_{\text{forcing .}\ t}\}$-sequences, where forcing is solar, orbital, volcanic, land use/land cover, well-mixed greenhouse gases, and the combination of all forcings (Arctic). All time series have the time unit of 10 years.

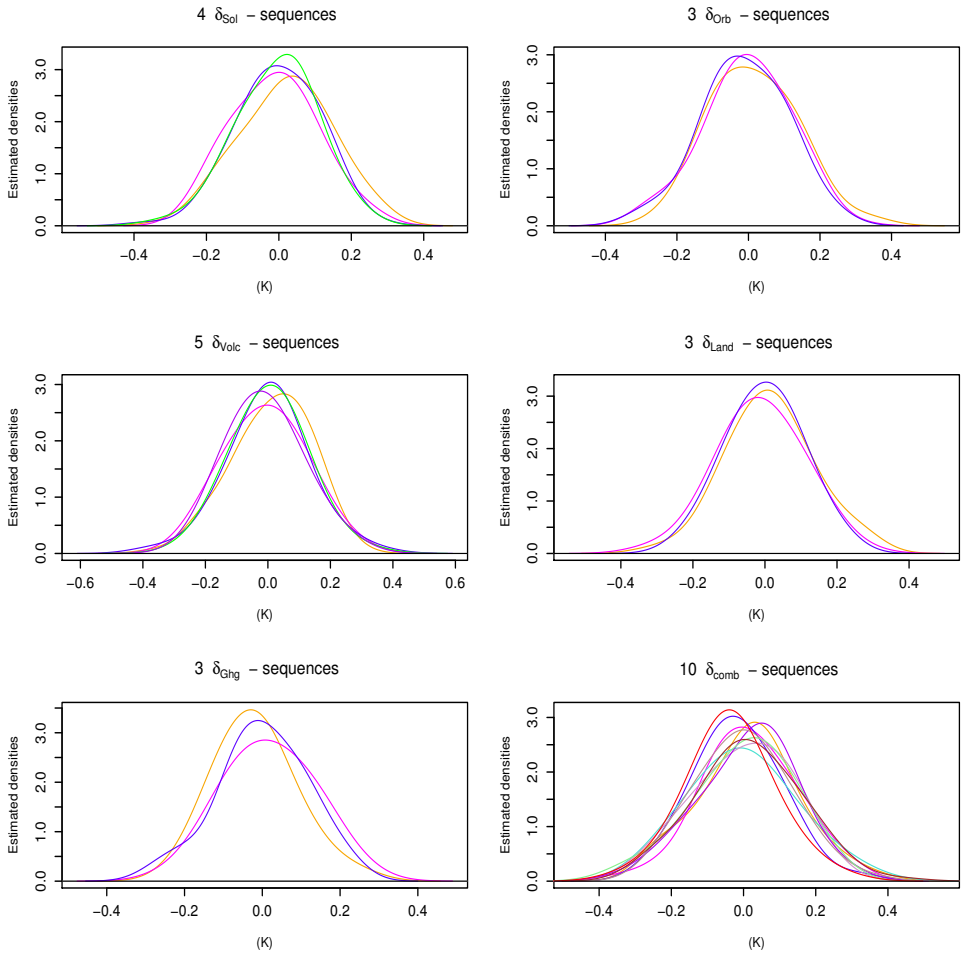

Figure S3.3. Estimated densities for the $\{\tilde{\delta}_{\text{forcing}\,t}\} = \{x_{\text{forcing, repl. i}\,t} - \bar{x}_{\text{forcing .}\ t}\}$-sequences, where forcing is solar, orbital, volcanic, land use/land cover, well-mixed greenhouse gases, and the combination of all forcings (North America). All time series have the time unit of 10 years.

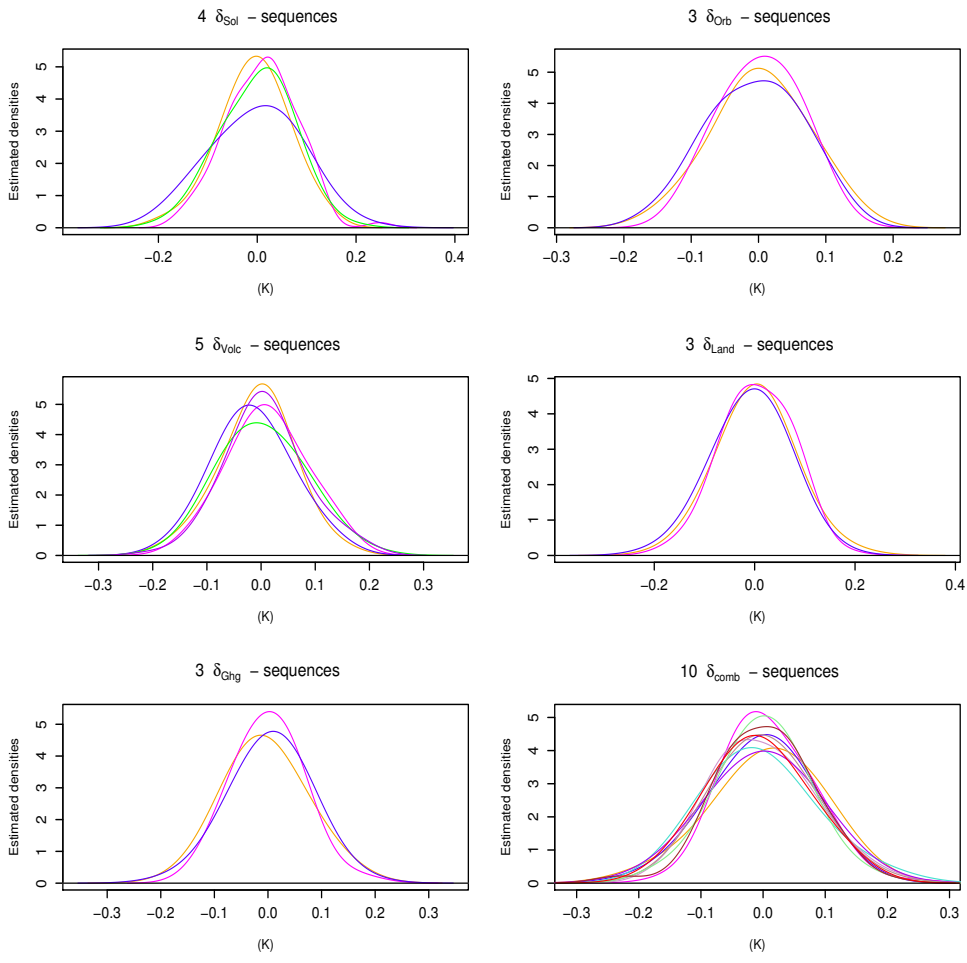

Figure S3.4. Estimated densities for the $\{\tilde{\delta}_{\text{forcing}\,t}\} = \{x_{\text{forcing, repl. i}\,t} - \bar{x}_{\text{forcing .}\ t}\}$-sequences, where forcing is solar, orbital, volcanic, land use/land cover, well-mixed greenhouse gases, and the combination of all forcings (Asia). All time series have the time unit of 10 years .

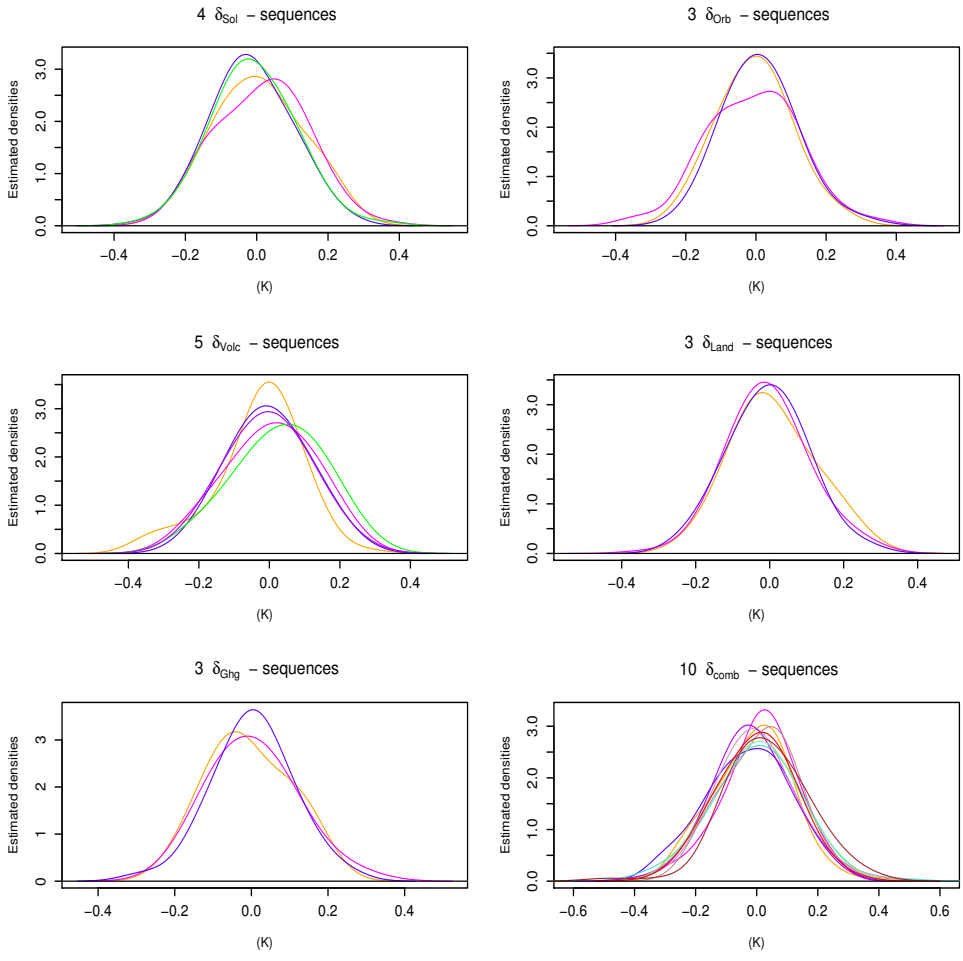

Figure S3.5. Estimated densities for the $\{\tilde{\delta}_{\text{forcing}\,t}\} = \{x_{\text{forcing, repl. i}\,t} - \bar{x}_{\text{forcing .}\,t}\}$-sequences, where forcing is solar, orbital, volcanic, land use/land cover, well-mixed greenhouse gases, and the combination of all forcings (South America). All time series have the time unit of 10 years.

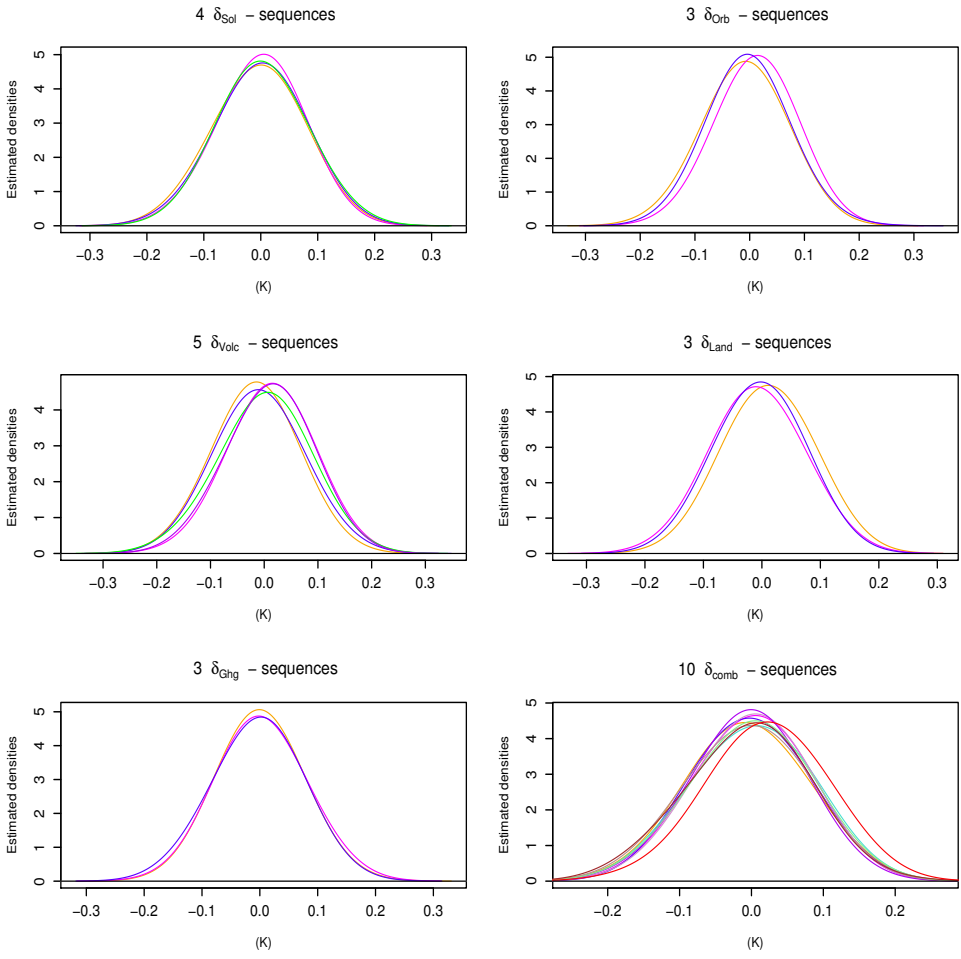

Figure S3.6. Estimated densities for the $\{\tilde{\delta}_{\text{forcing}\,t}\} = \{x_{\text{forcing, repl. i}\,t} - \bar{x}_{\text{forcing .}\ t}\}$-sequences, where forcing is solar, orbital, volcanic, land use/land cover, well-mixed greenhouse gases, and the combination of all forcings (Australasia). All time series have the time unit of 10 years.

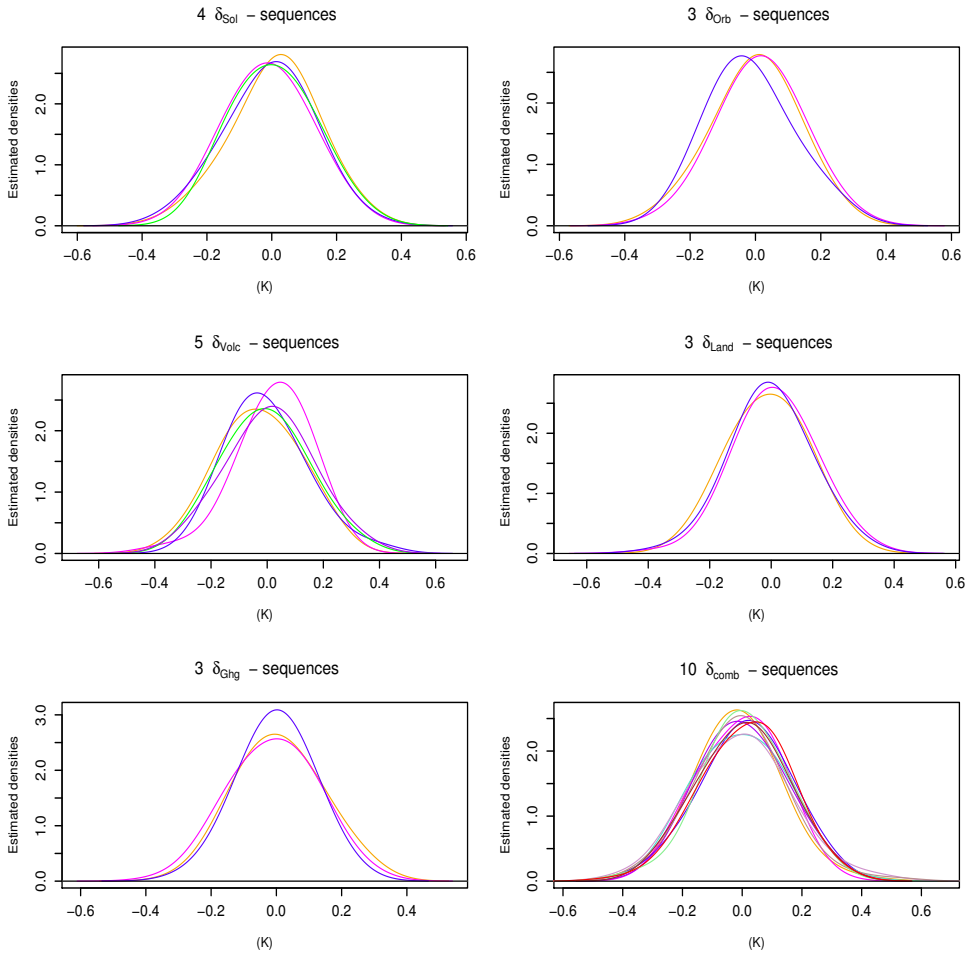

Figure S3.7. Estimated densities for the $\{\tilde{\delta}_{\text{forcing}\,t}\} = \{x_{\text{forcing, repl. i}\,t} - \bar{x}_{\text{forcing .}\,t}\}$-sequences, where forcing is solar, orbital, volcanic, land use/land cover, well-mixed greenhouse gases, and the combination of all forcings (Antarctica). All time series have the time unit of 10 years.

4 Forcing data

Here we show forcing data that are similar, though not identical, to the forcing data used in the CESM-LME simulations from which we obtained temperature data for our statistical analysis. The forcing data shown here were instead used in a similar, but unpublished, simulation experiment with the climate model EC-Earth v.3.1, which we have obtained through personal communication (September 2017) with Qiang Li, Department of Physical Geography, Stockholm University. EC-Earth v3.1 is a state-of-the-art Earth System Model of the same kind as CESM, but developed by a consortium of European research institutions (`http://www.ec-earth.org/`). The EC-Earth model has recently been run in a simulation of the last millennium with similar forcings as in CESM-LME.

Both CESM and EC-Earth were used with forcings following the protocols of PMIP3 (`https://pmip3.lsce.ipsl.fr/wiki/doku.php/pmip3:design:lm:final`) as discussed by [10], [11]. This protocol, however, made it possible for the modelling teams to choose between some agreed alternatives. This explains why forcing data are not identical in CESM and EC-Earth. The reason why we show the slightly different forcing data used with EC-Earth instead of those used in CESM, is simply that it made it easier for us, thanks to our close contact with the climate modeller Qiang Li, to obtain relevant forcing data specifically calculated over each of the seven regions/seasons used in our analysis.

The purpose of showing figures with forcing data is to provide a visual impression of the temporal evolution of the different forcings and their differences between regions. Although the forcings themselves are not included in our statistical models, the knowledge about their evolution can be helpful for forming conceptions about expected magnitudes of the model coefficients (factor loadings), which ultimately may help us to judge how reasonable numerical solutions suggested by our statistical models are (see [4], Sec. 3.1 for discussion).

In our opinion, the forcing data provided here is sufficiently similar to those used in the CESM-LME simulations in order to be used in our statistical analysis. Nevertheless, for the sake of completeness and transparency, we provide references to the forcing data sets used with both CESM (as described in [6] and [8] and EC-Earth (as explained to us by Qiang Li):

Total solar irradiance

CESM: Changes in total solar irradiance (TSI) were prescribed using the reconstruction of [13] (hereafter VSK) merged to [7] at 1834 to have a

smooth transition to twentieth-century simulations. The VSK reconstruction is based on physical modeling of the solar surface magnetic flux and its relationship with the isotopes. An estimated 11-yr solar cycle is superimposed on the VSK reconstruction as derived by [11]. The decrease in TSI at the Maunder Minimum period (i.e. 1645-1715; a period when sunspots were exceptionally rare) as compared to modern times is 0.1%.

EC-Earth: Changes in TSI are prescribed using the VSK values during 850-1609. Thereafter, values from [14] were used.

Greenhouse gases

CESM: The evolution of the concentrations of the principal long-lived greenhouse gases (CO2, CH4, and N2O) were imposed based on estimates from high-resolution ice cores in Antarctica. See [10], [11] for discussion and references. Small variations from 850 to about 1800 are related to natural feedbacks in the carbon and nitrogen cycles. Post-1800, anthropogenic changes in these GHGs are apparent.

EC-Earth: The table provided through PMIP3 at
`https://wiki.lsce.ipsl.fr/pmip3/lib/exe/fetch.php/pmip3:design:lm:ghg_lawdome_giss_merge_c5mip_24jul09.1-2000.txt`
was used. This should be the same as for CESM.

Orbital parameters

CESM and EC-Earth: Seasonal and latitudinal distribution of the orbital modulation of insolation from the equations in [1] was used.

Volcanic forcing

CESM: The ice core–based index of [3], which is based on volcanic deposition signals from ice core records from both the Arctic and Antarctica, was adopted. This reconstruction provides estimates of the stratospheric sulfate aerosol loadings from volcanic eruptions, modeled as a function of latitude, altitude, and month. Stratospheric aerosols are prescribed in the model as a fixed single-size distribution in the three layers in the lower stratosphere above the tropopause.

EC-Earth: The data set from [2] was used. Aerosol optical depth (AOD) estimates are obtained based on a correlation between sulphate in Antarctic ice cores and satellite data. The time resolution of the series is ten days and the data are provided at four equal area latitude bands. One difference between the two volcanic forcing reconstructions is that some eruptions with very high AOD values in the reconstruction from [3] have significantly

smaller corresponding AOD values in the reconstruction from [2]. Another difference is that the exact year when eruptions occur differ between the two reconstructions in some cases. An illustration of these differences is presented, at a global scale, in Figure 4 in the discussion by [10].

Land-use

CESM: The reconstruction of land use from [9] was merged with that of [5] to give a consistent and smoothly evolving land use change. The datasets were spliced together by scaling the dataset of [9] to match the dataset of [5] at 1500 for every land model grid point. The only plant functional types (PFTs) that were changed are those for crops and pasture; all other PFTs remain at their 1850 control prescription.

EC-Earth: The reconstruction of land use from [9] was adopted. A description of the dataset is provided at

`https://cera-www.dkrz.de/WDCC/ui/Compact.jsp?acronym=RECON_LAND_COVER_800-1992.`

The dataset distinguishes between C3 pasture and C4 pasture in the anthropogenically influenced areas.

In Figures S4.1-5, the forcing data are shown graphically.

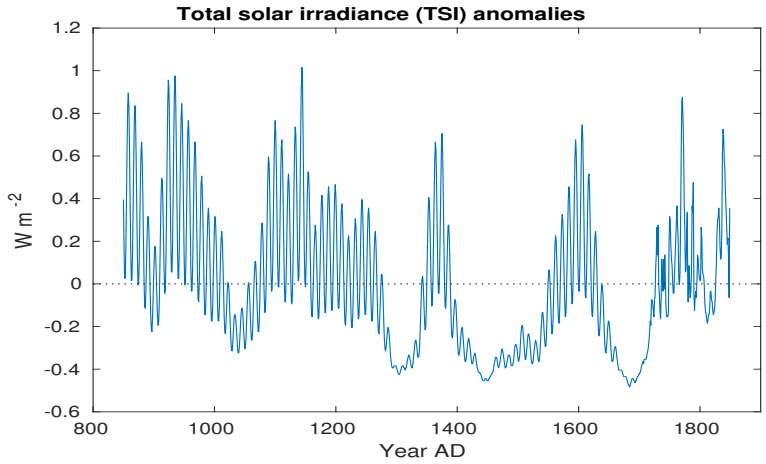

Figure S4.1. Solar forcing data; i.e. Total solar irradiance shown as anomalies from the long-term average. The time series has the time unit of 1 year.

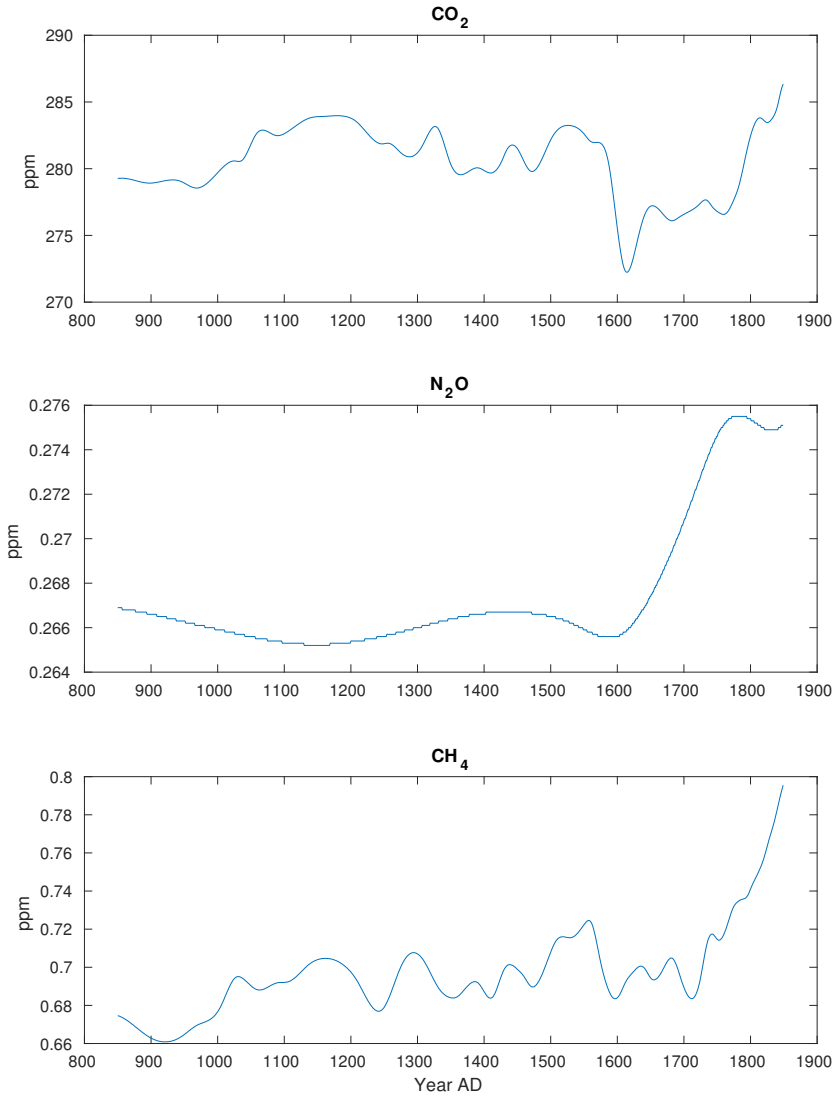

Figure S4.2. Well-mixed greenhouse gas concentrations. All time series have the time unit of 1 year.

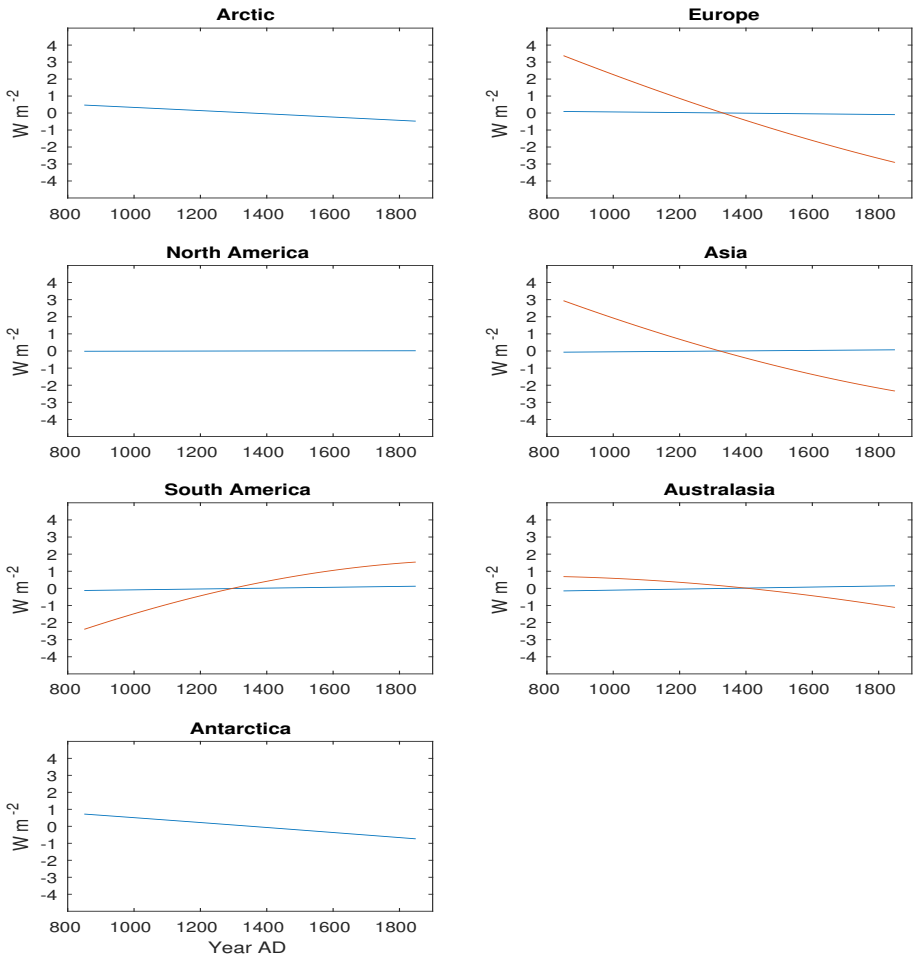

Figure S4.3. Orbital forcing; i.e. the orbital modulation of incoming solar radiation per latitude and time. The annual average value is shown in blue for all regions. For the regions where seasonal mean temperatures are used in our analysis, the corresponding seasonal forcing values are shown in red; i.e. Europe - JJA, Asia - JJA, South America - DJF, Australasia - Sep-Feb. All time series are shown as anomalies from their long-term averages and have the time unit of 1 year.

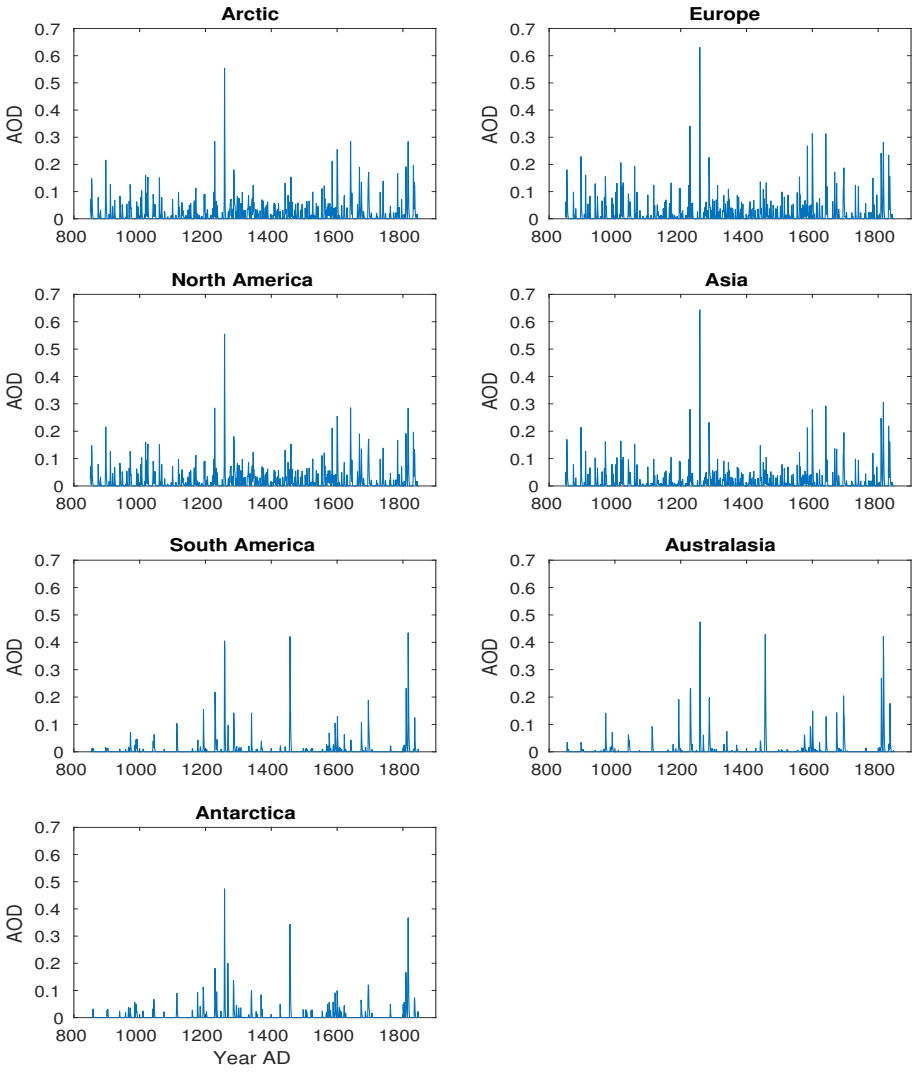

Figure S4.4. Volcanic forcing data; presented as Aerosol Optical Depth approximately averaged over the respective latitude band and season for each region. All time series have the time unit of 1 year.

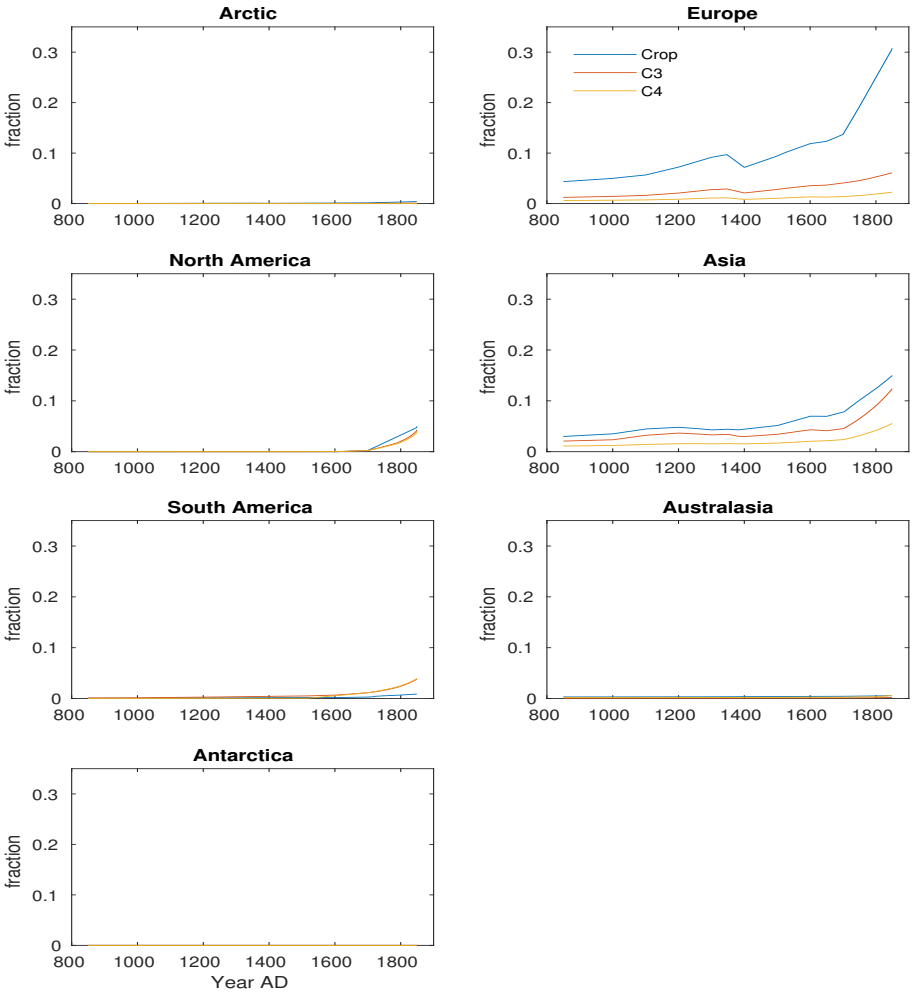

Figure S4.5. Land use/land cover forcing data; presented as the fraction of the land area in each region that is covered by Crops and C3- and C4-vegetation. Note 1: The land use/land cover forcing data used in the CESM-LME simulations did only consider changes in Crop and Pasture, whereas this figure shows a somewhat different dataset which distinguishes between Crops and C3- and C4-vegetation. Note 2: The regional temperature data from CESM-LME used in our analysis include sea surface temperatures in the three regions of the Arctic, North America, and Australasia. Hence, the numbers plotted here for these three regions are larger

than if the averaging had been made over the entire areas of the corresponding land+ocean regions. All time series have the time unit of 1 year.

5 Relationships between the U_R- and U_T-statistics: a Monte Carlo experiment

Consider the following *mean-centered* variables, defined in [12]:

$$\begin{aligned} &\bullet \quad x_{\mathrm{u}\,t} = \delta_{\mathrm{u}\,t}, \\ &\bullet \quad x_{\mathrm{f}\,t} = \alpha \cdot \xi_{\mathrm{f}\,t} + \tilde{\delta}_{\mathrm{f}} \\ &\bullet \quad z_t \approx \tau_t + \epsilon_t = \xi_{\mathrm{f}\,t} + \eta_t + \epsilon_t \end{aligned} \tag{S5.1}$$

where
$x_{\mathrm{u}\,t}$ - a simulated temperature value, generated by an unforced climate model, for the region of interest and time period of t, $t = 1, 2, \ldots, n$;

$\delta_{\mathrm{u}\,t}$ - the internal random variability of the unforced climate model;

$x_{\mathrm{f}\,t}$ - a simulated temperature value, generated by a forced climate model, corresponding to $x_{\mathrm{u}\,t}$;

$\xi_{\mathrm{f}\,t}$ - the true effect of a specific type of forcing that has influenced the true temperature τ_t;

$\tilde{\delta}_{\mathrm{f}\,t}$ - the internal random variability of the forced climate model;
z_t - a properly calibrated proxy;

τ_t - an unobservable true temperature;

η_t - the residual variation in true temperature that cannot be statistically explained by the particular forcing under consideration;

ϵ_t - the residual non-climatic variation in z_t.

Notice that despite the same notation, α in (S5.1) corresponds to β in the ME model from (2.4) in Part I, rather than to α in the FA(2,1)-model from (4.1.4) in Part I. The link between these statistical models and their parameters is explained in Sec. 5 in Part I.

To illustrate the relationships between the U_R- and U_T-test statistics, defined in Appendix A, we generate artificial sets of data. In this study, all variables in (S5.1) are assumed to have the characteristics of white noise[1].

[1] A sequence of uncorrelated random variables, each with zero mean and variance σ^2 is referred to as **white noise**.

Further, for our analysis in Part III, it is sufficient to focus attention here on studying artificial data satisfying $\sigma_\epsilon^2 = 0$. The procedure of generating data is the following.

For each iteration step i, $i = 1, 2, \ldots, 1000$, generate:

1. 100 independent values of $\xi_{\mathsf{f}} \sim \mathrm{N}(0, \sigma_\xi^2)$. The variance of ξ_{f} comprises either 50% or 25% of the variance of the artificial τ, which amounts to modelling a (relatively) strong and (relatively) weak forced signal.
2. 100 independent values of $\delta_{\mathsf{u}} \sim \mathrm{N}(0, \sigma_{\delta_{\mathsf{u}}}^2)$.
3. 100 independent values of $\tilde{\delta}_{\mathsf{f\,repl.\ j}} \sim \mathrm{N}(0, \sigma_{\delta_{\mathsf{u}}}^2)$, $j = 1, 2, 3$.
4. 100 independent values of $\eta \sim \mathrm{N}(0, \sigma_\eta^2)$.
5. Form the data

$$\begin{pmatrix} x_{\mathsf{u}t} \\ x_{\mathsf{f\,repl.\ 1}\,t} \\ x_{\mathsf{f\,repl.\ 2}\,t} \\ x_{\mathsf{f\,repl.\ 3}\,t} \\ \tau_t \end{pmatrix} = \begin{pmatrix} & & \tilde{\delta}_{\mathsf{u}} t \\ \alpha \cdot \xi_{\mathsf{f}\,t} & + & \tilde{\delta}_{\mathsf{f\,repl.1}} t \\ \alpha \cdot \xi_{\mathsf{f}\,t} & + & \tilde{\delta}_{\mathsf{f\,repl.2}} t \\ \alpha \cdot \xi_{\mathsf{f}\,t} & + & \tilde{\delta}_{\mathsf{f\,repl.3}} t \\ \xi_{\mathsf{f}\,t} & + & \eta_t \end{pmatrix}$$

   where $t = 1, 2, \ldots, 100$, and $\alpha = 0$, 0.5, 1, 1.5, 2, or 2.5.

7. Build the mean-sequence $\bar{x}_{\mathsf{f}}$ for calculating the U_R-statistic.
8. Calculate the U_R- and U_T-statistics. Both statistics are unweighted, meaning that $w_t = 1$.

The result is presented graphically in Figures S5.1 - S5.2.

Case I: Strong forced signal, i.e. the variance of ξ_f comprises 50% of the variance of the artificial τ.

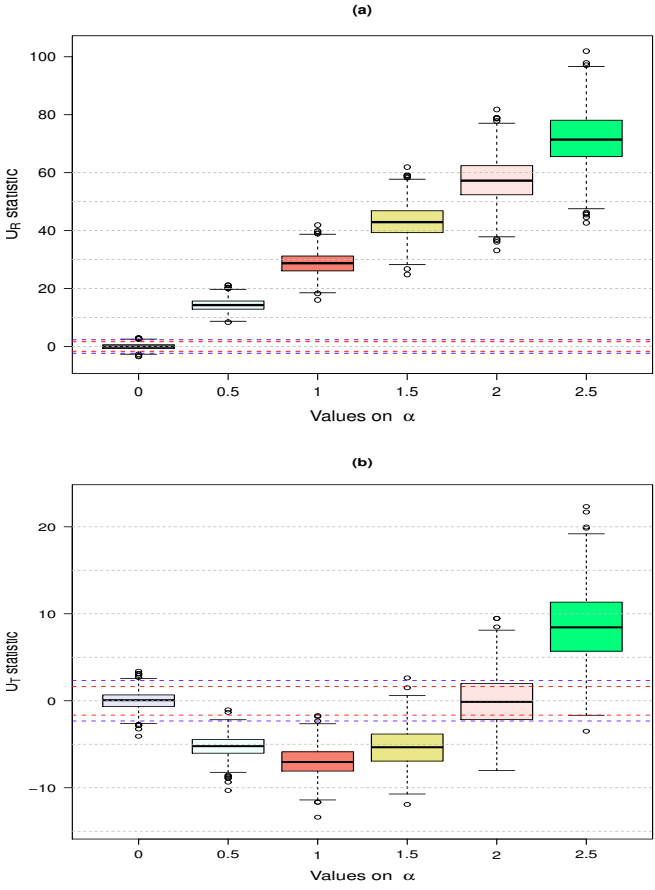

Figure S5.1. Boxplots for: (a) the correlation $U_R(\bar{x}_f, \tau)$- test statistic, and (b) the distance-based $U_T(x_{\text{frepl. 1}}, x_{\text{frepl. 2}}, x_{\text{frepl. 3}}, x_u, \tau)$ statistic (both unweighted). The variance of ϵ is zero.

Case II: weak forced signal, i.e. the variance of ξ_{f} comprises 25% of the variance of the artificial τ.

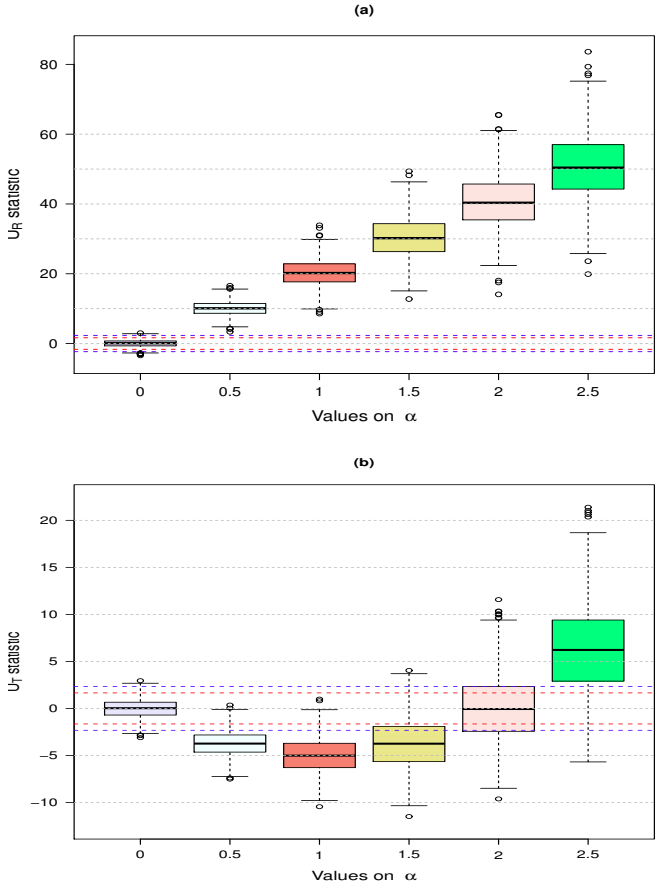

Figure S5.2. Boxplots for the correlation $U_R(\bar{x}_{\mathrm{f}}, \tau)$- test statistic, and for the distance-based $U_T(x_{\mathrm{f}}, x_{\mathrm{u}}, \tau)$ statistic (both unweighted). The variance of ϵ is zero.

6 Tables

6.1 Europe (summer, June-August, mean temperature)

Table S6.1.1. *The results of fitting the FA(2,1)-model and calculating the U_R- and U_T-test statistics. The results for the FA(2,1)-model and the U_R-statistic are based on the data sets $\{\bar{x}_{\text{single forcing}}, \bar{x}^{(i)}_{comb}\}$, where $\bar{x}^{(i)}_{comb}$ belongs to Data set number i, $i = 1, 2, \ldots, 10$, each of which is described in Table 1. The results for the U_T-statistic are based on the data sets $\{x_{\text{single forcing, repl. 1}}, \ldots, x_{\text{single forcing, repl. k}}, x_u, \bar{x}^{(i)}_{comb}\}$, where x_u is the unforced climate model simulation*

- Single forcing: *Solar*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.038	< 0.01	0.027	< 0.01	0.23	0.63	0.99	0.99	0.029	0.74	-0.023

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$.

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.038	0.024	0.001	0.62	0.998	0.993	0.002	0.674	-0.416
Mean	0.041	0.026	0.071	0.844	0.999	0.998	0.012	1.295	-0.191
Max	0.043	0.029	0.244	0.98	1.000	1.000	0.030	1.918	0.036

- Single forcing: *Orbital*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.045	< 0.01	0.026	< 0.01	0.69	0.40	0.99	0.98	0.053	2.10	-0.72

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$.

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.0453	0.0224	0.70	0.041	0.959	0.878	0.053	2.10	-1.373
Mean	0.0502	0.0245	2.35	0.170	0.977	0.930	0.094	2.90	-1.020
Max	0.0542	0.0276	4.19	0.403	0.993	0.979	0.129	3.56	-0.497

- Single forcing: *Volcanic*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.134	< 0.01	0.008	< 0.01	3.41	0.06	0.97	0.90	0.086	15.97	-2.76

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$.

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.130	0.0068	1.13	0.003	0.922	0.767	0.051	15.5	-3.13
Mean	0.134	0.0074	3.53	0.107	0.966	0.899	0.084	16.0	-2.70
Max	0.139	0.0089	8.54	0.288	0.989	0.966	0.129	16.7	-2.23

- Single forcing: *Land*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.055	< 0.01	0.024	< 0.01	6.43	0.011	0.94	0.82	0.17	3.68	-0.839

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$.

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.051	0.020	3.93	0.002	0.913	0.740	0.131	3.04	-1.107
Mean	0.055	0.023	6.53	0.016	0.938	0.814	0.167	3.67	-0.797
Max	0.059	0.026	9.37	0.047	0.962	0.884	0.201	4.23	-0.347

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- Single forcing: *Greenhouse gases (Ghg)*, $i = 10$

$\widehat{\alpha}$	p-value	$\widehat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.054	< 0.01	0.035	< 0.01	0.158	0.69	0.99	0.99	0.025	2.30	-0.487

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$.

	$\widehat{\alpha}$	$\widehat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.0523	0.022	0.036	0.550	0.996	0.989	0.012	1.95	-0.603
Mean	0.0540	0.024	0.194	0.680	0.998	0.994	0.026	2.34	-0.460
Max	0.0554	0.028	0.357	0.849	1.000	0.999	0.037	2.59	-0.206

Table S6.1.2. *The final factor model obtained by simplifying the basic FA(6,6)-model given in Table 2 (Europe)*

- *The result for Data set number 10*

Parameter	Estimate	p-value
Ssim	0.022	9.3e-02
Osim	0.029	2.5e-02
Vsim	0.129	5.4e-33
Lsim	0.038	3.6e-03
Gsim	0.042	2.9e-04
ϕ_{LG}	0.329	5.5e-01

To assess the overall model fit

Model $\chi^2 = 24.85$ df $= 15$ p-value $= 0.052$

$GFI = 0.92$, $AGFI = 0.88$, $SRMR = 0.1285$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x_{comb}
x_{Sol}	1.43	1.50	0.77	0.68	2.25	0.34
x_{Orb}		0.54	1.59	2.01	0.70	1.51
x_{Volc}			-0.38	2.40	0.72	0.65
x_{Land}				-0.41	0.19	2.33
x_{Ghg}					0.59	0.67
x_{comb}						0.76

- *Summary of the results based on all 10 data sets*

	Min	Mean	Max
$\widehat{\text{Ssim}}$	0.022	0.028	0.032
$\widehat{\text{Osim}}$	0.029	0.036	0.044
$\widehat{\text{Vsim}}$	0.126	0.130	0.132
$\widehat{\text{Lsim}}$	0.027	0.035	0.039
$\widehat{\text{Gsim}}$	0.040	0.0342	0.045
ϕ_{LG}	0.017	0.28	0.33
Model χ^2	22	25	32
p-value	0.01	0.06	0.12
GFI	0.90	0.92	0.92
AGFI	0.86	0.88	0.89
SRMR	0.125	0.131	0.140

The solution for each data set is proper.

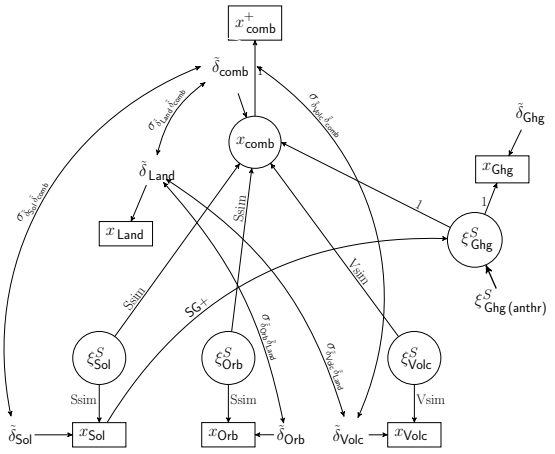

Figure S6.1.1. *Path diagram of the final SEM model fitted to the* Europe *data in Step 1.*

Table S6.1.3. *The result of estimating the SEM model presented graphically in Figure S6.1.1*

- *The result based on Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.039	5.4e-07	$\sigma_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{comb}}}$	0.0047	4.8e-04
Vsim	0.127	7.4e-29	$\sigma_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{comb}}}$	-0.0031	0.015
SG+	0.232	0.053	$\sigma_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{comb}}}$	0.0019	0.036
$\sigma_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{Land}}}$	0.029	0.025	$\text{Var}(\xi^S_{\text{Ghg (anthr)}})$	0.0023	0.012
$\sigma_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Land}}}$	0.0015	0.07			

To assess the overall model fit

Model $\chi^2 = 6.1$ df $= 12$ p-value $= 0.9$

$GFI = 0.98$ $AGFI = 0.96$ $SRMR = 0.072$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}
x_{Sol}	0.18	1.33	0.73	0.68	0.09	0.70
x_{Orb}		-0.03	1.56	0.37	0.65	0.99
x_{Volc}			-0.09	0.49	0.71	0.32
x_{Land}				0.71	0.77	0.64
x_{Ghg}					-0.08	0.63
x^+_{comb}						0.56

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• *Summary of the results based on all data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\text{Ssim}}$	0.037	0.042	0.047	$\widehat{\sigma}_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{comb}}}$	3.8e-03	4.7e-03	5.7e-03
$\widehat{\text{Vsim}}$	0.124	0.125	0.128	$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{comb}}}$	-3.6e-03	-2.9e-03	-2.2e-03
$\widehat{\text{SG+}}$	0.226	0.236	0.242	$\widehat{\sigma}_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{comb}}}$	1.8e-03	2.3e-03	3.1e-03
$\widehat{\sigma}_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{Land}}}$	2.9e-03	3.0e-03	3.1e-03	$\widehat{\text{Var}}(\xi^S_{\text{Ghg (anthr)}})$	1.9e-03	2.2e-03	2.5e-03
$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Land}}}$	1.2e-03	1.4e-03	1.5e-03				
Model χ^2	5.7	6.6	7.8	GFI	0.98	0.98	0.98
p-value	0.80	0.88	0.93	AGFI	0.96	0.96	0.97
SRMR	0.072	0.077	0.085				

The solution for each data set is proper

Under the SEM model in Figure S6.1.1, $\text{Var}(\xi^S_{\text{Ghg}}) = (\text{SG+})^2 \cdot \text{Ssim}^2 + \sigma^{2\,*}_{\tilde{\delta}_{\text{Sol}}} \cdot (\text{SG+})^2 + \text{Var}(\xi^S_{\text{Ghg (anthr)}})$, where $\sigma^{2\,*}_{\tilde{\delta}_{\text{Sol}}}$ is known a priori. For Data set number 10, $\widehat{\text{Var}}(\xi^S_{\text{Ghg}}) = 0.0027$ with the associated p-value 0.0083. Hence, the stand. coefficient for ξ^S_{Ghg} is estimated to be as large as $\widehat{\text{Gsim}} = \sqrt{\widehat{\text{Var}}(\xi^S_{\text{Ghg}})} = 0.052$, associated with the same p-value of 0.0083.

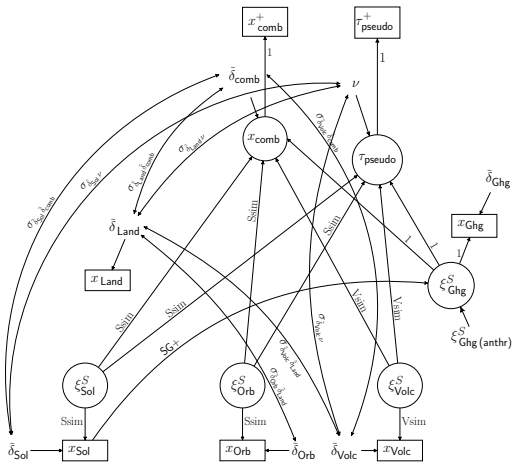

Figure S6.1.2. *Path diagram of the final SEM model fitted to the* Europe *data in Step 2.*

Table S6.1.4. *The result of estimating the SEM model presented graphically in Figure S6.1.2*

- *The result based on Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.040	4.1e-06	$\sigma_{\tilde{\delta}_{\mathsf{Land}}\,\tilde{\delta}_{\mathsf{comb}}}$	4.8e.03	4.5e-04
Vsim	0.128	3.2e-27	$\sigma_{\tilde{\delta}_{\mathsf{Sol}}\,\tilde{\delta}_{\mathsf{comb}}}$	-3.0e-03	0.011
SG+	0.235	0.046	$\sigma_{\tilde{\delta}_{\mathsf{Volc}}\,\tilde{\delta}_{\mathsf{comb}}}$	1.8e-03	0.047
$\mathrm{Var}(\xi^S_{\mathsf{Ghg\,(anthr)}})$	0.0021	0.018	$\sigma_{\tilde{\delta}_{\mathsf{Sol}}\,\nu}$	-1.6e-03	0.39
$\sigma_{\tilde{\delta}_{\mathsf{Volc}}\,\tilde{\delta}_{\mathsf{Land}}}$	2.9e-03	0.025	$\sigma_{\tilde{\delta}_{\mathsf{Volc}}\,\nu}$	1.0e-03	0.58
$\sigma_{\tilde{\delta}_{\mathsf{Orb}}\,\tilde{\delta}_{\mathsf{Land}}}$	1.5e-03	0.065	$\sigma_{\tilde{\delta}_{\mathsf{Land}}\,\nu}$	4.2e-03	0.034
σ^2_ν	0.030	6.0e-10			

To assess the overall model fit

Model $\chi^2 = 10$ df $= 15$ p-value $= 0.8$

$GFI = 0.97$ $AGFI = 0.95$ $SRMR = 0.075$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}	τ^+_{pseudo}
x_{Sol}	0.08	1.32	0.72	0.68	0.04	0.74	0.40
x_{Orb}		-0.11	1.54	0.34	0.65	0.91	1.08
x_{Volc}			-0.19	0.48	0.71	0.24	0.73
x_{Land}				0.71	0.77	0.61	0.75
x_{Ghg}					0.06	0.77	0.95
x^+_{comb}						0.49	0.95
τ^+_{pseudo}							0.70

- *Summary of the results based on all data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\mathrm{Ssim}}$	0.038	0.043	0.047	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Land}}\,\tilde{\delta}_{\mathsf{comb}}}$	3.8e-03	4.8e-03	5.6e-03
$\widehat{\mathrm{Vsim}}$	0.123	0.125	0.128	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Sol}}\,\tilde{\delta}_{\mathsf{comb}}}$	-3.5e-03	-2.9e-03	-2.1e-03
$\widehat{\mathrm{SG+}}$	0.223	0.234	0.241	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Volc}}\,\tilde{\delta}_{\mathsf{comb}}}$	1.8e-03	2.3e-03	3.1e-03
$\widehat{\mathrm{Var}}(\xi^S_{\mathsf{Ghg\,(anthr)}})$	2.0e-03	2.2e-03	2.4e-03	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Sol}}\,\nu}$	-6.0e-03	-2.9e-03	-1.5e-03
$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Volc}}\,\tilde{\delta}_{\mathsf{Land}}}$	2.8e-03	3.0e-03	3.2e-03	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Volc}}\,\nu}$	1.0e-03	2.2e-03	4.8e-03
$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Orb}}\,\tilde{\delta}_{\mathsf{Land}}}$	1.2e-03	1.4e-03	1.5e-03	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Land}}\,\nu}$	2.8e-03	4.7e-03	8.0e-03
$\widehat{\sigma}^2_\nu$	0.019	0.025	0.034				
Model χ^2	6.8	9.8	14.4	GFI	0.96	0.97	0.98
p-value	0.50	0.81	0.96	AGFI	0.93	0.95	0.96
SRMR	0.069	0.079	0.091				

The solution for each data set is proper

6.2 Arctic (annual mean temperature)

Table S6.2.1. *The results of fitting the FA(2,1)-model and calculating the U_R- and U_T-test statistics. The results for the FA(2,1)-model and the U_R-statistic are based on the data sets $\{\bar{x}_{\text{single forcing}}, \bar{x}^{(i)}_{comb}\}$, where $\bar{x}^{(i)}_{comb}$ belongs to Data set number i, $i = 1, 2, \ldots, 10$, each of which is described in Table 1. The results for the U_T-statistic are based on the data sets $\{x_{\text{single forcing, repl. 1}}, \ldots, x_{\text{single forcing, repl. k}}, x_u, \bar{x}^{(i)}_{comb}\}$, where x_u is the unforced climate model simulation*

- Single forcing: *Solar*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.105	< 0.01	0.062	< 0.01	3.98	0.046	0.96	0.88	0.12	4.38	-1.28

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.0972	0.0579	1.40	0.014	0.94	0.83	0.073	3.50	-1.62
Mean	0.1028	0.0616	3.29	0.100	0.97	0.90	0.109	4.13	-1.05
Max	0.1085	0.0691	6.11	0.237	0.99	0.96	0.149	4.80	-0.42

- Single forcing: *Orbital*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.121	< 0.01	0.058	< 0.01	3.69	0.054	0.96	0.89	0.122	4.42	-1.205

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.110	0.051	1.36	0.03	0.96	0.87	0.074	3.45	-1.30
Mean	0.118	0.059	2.90	0.11	0.98	0.91	0.107	4.13	-0.97
Max	0.125	0.070	4.53	0.24	0.99	0.96	0.137	4.76	-0.42

- Single forcing: *Volcanic*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.208	< 0.01	0.029	< 0.01	1.46	0.23	0.99	0.96	0.062	13.07	-2.428

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.205	0.024	0.68	0.10	0.97	0.92	0.043	12.8	-2.85
Mean	0.207	0.028	1.37	0.27	0.99	0.96	0.059	13.1	-2.28
Max	0.210	0.035	2.64	0.41	0.99	0.98	0.083	13.5	-1.85

- Single forcing: *Land*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.096	< 0.01	0.066	< 0.01	3.09	0.079	0.97	0.91	0.12	2.89	-0.138

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.063	0.060	0.11	0.05	0.96	0.89	0.022	1.00	-0.26
Mean	0.085	0.067	1.84	0.26	0.98	0.95	0.087	2.27	0.22
Max	0.100	0.080	3.85	0.75	0.99	0.99	0.135	3.19	0.94

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- Single forcing: *Greenhouse gases (Ghg)*, $i = 10$

$\widehat{\alpha}$	p-value	$\widehat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.099	< 0.01	0.69	< 0.01	0.64	0.42	0.99	0.98	0.049	1.60	-0.33

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$

	$\widehat{\alpha}$	$\widehat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.094	0.061	0.029	0.24	0.986	0.96	0.011	1.11	-0.81
Mean	0.099	0.068	0.624	0.48	0.994	0.98	0.045	1.71	-0.25
Max	0.108	0.077	1.384	0.86	1.000	0.99	0.072	2.36	-0.18

Table S6.2.2. *The final FA-model obtained by simplifying the basic FA(6,6)-model in Table 2 (the Arctic data)*

- *The result for Data set number 10*

Parameter	Estimate	p-value
Ssim	0.0824	1.0e-06
Osim	0.0934	5.2e-07
Vsim	0.198	1.3e-28
Gsim	0.071	4.5e-04

To assess the overall model fit

Model $\chi^2 = 38.12$ df $= 17$ p-value $= 0.0024$

$GFI = 0.87$, $AGFI = 0.84$, $SRMR = 0.162$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}
x_{Sol}	0.07	2.51	1.84	1.49	1.98	2.23
x_{Orb}		-0.03	1.76	0.79	0.53	2.18
x_{Volc}			0.00	3.26	0.69	0.82
x_{Land}				0.61	1.88	2.92
x_{Ghg}					1.69	0.41
x^+_{comb}						0.12

- *Summary of the results based on all 10 data sets*

	Min	Mean	Max
$\widehat{\mathrm{Ssim}}$	0.044	0.085	0.092
$\widehat{\mathrm{Osim}}$	0.053	0.076	0.098
$\widehat{\mathrm{Vsim}}$	0.196	0.199	0.202
$\widehat{\mathrm{Gsim}}$	0.041	0.076	0.087
Model χ^2	35	37	41
P-value	0.001	0.006	0.007
GFI	0.87	0.88	0.88
AGFI	0.83	0.85	0.86
SRMR	0.147	0.156	0.165

The solution for each data set is proper

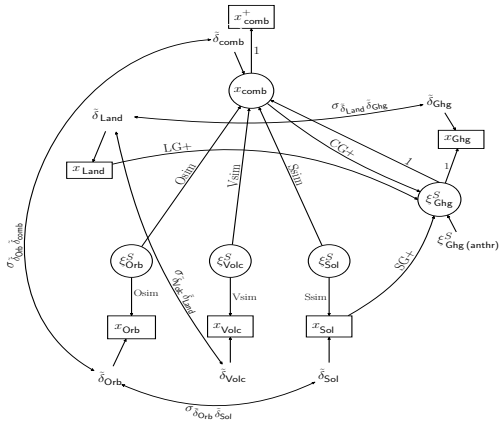

Figure S6.2.1. *Path diagram of the final SEM model fitted to the* Arctic *data in Step 1.*

Table S6.2.3. *The result of estimating the SEM model presented graphically in Figure S6.2.1*

- *The result based on Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.081	7.3e-06	$\sigma_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{Land}}}$	0.014	5.3e-05
Osim	0.087	4.1e-04	$\sigma_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Sol}}}$	7.3e-03	0.014
Vsim	0.199	3.2e-29	$\sigma_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{comb}}}$	0.007	0.08
SG+	0.256	0.019	$\sigma_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{Ghg}}}$	-0.0075	0.13
LG+	0.424	8.0e-03	$\text{Var}(\xi^S_{\text{Ghg (anthr)}})$	0.011	0.07
CG+	-0.143	0.11			

To assess the overall model fit

Model $\chi^2 = 7.53$ df $= 10$ p-value $= 0.67$

$GFI = 0.98$ $AGFI = 0.95$ $SRMR = 0.087$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}
x_{Sol}	0.13	0.11	1.84	1.50	0.19	1.23
x_{Orb}		0.21	1.78	0.81	0.55	1.18
x_{Volc}			-0.08	-0.04	0.59	0.73
x_{Land}				0.61	0.36	0.51
x_{Ghg}					0.55	0.44
x^+_{comb}						0.68

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• *Summary of the results based on all 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\text{Ssim}}$	0.067	0.079	0.088	$\widehat{\sigma}_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{Land}}}$	0.014	0.014	0.015
$\widehat{\text{Osim}}$	0.077	0.086	0.096	$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Sol}}}$	0.007	0.007	0.008
$\widehat{\text{Vsim}}$	0.198	0.201	0.203	$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{comb}}}$	0.002	0.005	0.009
$\widehat{\text{SG+}}$	0.194	0.233	0.264	$\widehat{\sigma}_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{Ghg}}}$	-0.0084	-0.0054	-0.0016
$\widehat{\text{LG+}}$	0.180	0.340	0.470	$\widehat{\text{Var}}(\xi^S_{\text{Ghg (anthr)}})$	0.008	0.010	0.0130
$\widehat{\text{CG+}}$	-0.159	-0.112	-0.068				
Model χ^2	7.3	8.0	8.8	GFI	0.97	0.97	0.98
p-value	0.55	0.63	0.70	AGFI	0.94	0.95	0.95
SRMR	0.084	0.087	0.091				

The solution for each data set was proper.

Under the SEM model in Figure S6.2.1, $\text{Var}(\xi^S_{\text{Ghg}})$ is given by

$$\begin{aligned}\text{Var}(\xi^S_{\text{Ghg}}) = &\Big((\text{CG+})^2 \cdot \text{Osim}^2 + (\text{CG+})^2 \cdot \text{Ssim}^2 + (\text{CG+})^2 \cdot \text{Vsim}^2 + \sigma^{2\,*}_{\text{comb}} \cdot (\text{CG+})^2 + \\ &+ 2 \cdot (\text{CG+}) \cdot (\text{SG+}) \cdot \text{Ssim}^2 + \sigma^{2\,*}_{\text{Land}} \cdot (\text{LG+})^2 + 2 \cdot \sigma_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{Ghg}}} \cdot (\text{LG+}) + \\ &+ (\text{SG+})^2 \cdot \text{Ssim}^2 + \sigma^{2\,*}_{\text{Sol}} \cdot (\text{SG+})^2 + \text{Var}(\xi^S_{\text{Ghg (anthr)}})\Big) \Big/ \Big((\text{CG+}) - 1\Big)^2.\end{aligned}$$

For Data set number 10, $\widehat{\text{Var}}(\xi^S_{\text{Ghg}}) = 0.01$ with the associated p-value $= 0.26$. Thus, the standardised coefficient for ξ^S_{Ghg} is estimated to be as large as $\sqrt{\widehat{\text{Var}}(\xi^S_{\text{Ghg}})} = 0.1$, with the same p-value of 0.26.

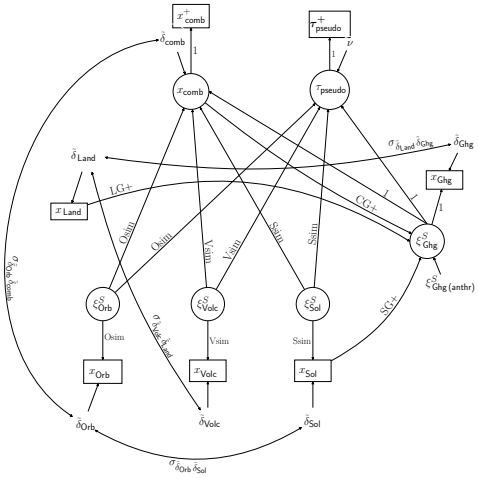

Figure S6.2.2. *Path diagram of the final SEM model fitted to the* Arctic *data in Step 2.*

Table S6.2.4. *The result of estimating the SEM model presented graphically in Figure S6.2.2*

- *The result based on Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.079	9.4e-06	$\sigma_{\tilde{\delta}_{\mathsf{Volc}}\,\tilde{\delta}_{\mathsf{Land}}}$	0.014	2.4e-05
Osim	0.098	2.5e-06	$\sigma_{\tilde{\delta}_{\mathsf{Orb}}\,\tilde{\delta}_{\mathsf{Sol}}}$	7.3e-03	0.014
Vsim	0.200	1.4e-29	$\sigma_{\tilde{\delta}_{\mathsf{Orb}}\,\tilde{\delta}_{\mathsf{comb}}}$	0.005	0.19
SG+	0.270	0.015	$\sigma_{\tilde{\delta}_{\mathsf{Land}}\,\tilde{\delta}_{\mathsf{Ghg}}}$	-7.3e-03	0.13
LG+	0.421	8.0e-03	$\mathrm{Var}(\xi^S_{\mathsf{Ghg\,(anthr)}})$	0.012	0.053
CG+	-0.154	0.08	σ^2_ν	0.077	7.2e-10

To assess the overall model fit

Model $\chi^2 = 10.62$ df $= 16$ p-value $= 0.83$

$GFI = 0.97$ $AGFI = 0.95$ $SRMR = 0.090$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}	τ^+_{pseudo}
x_{Sol}	0.22	0.08	1.84	1.51	0.13	1.26	1.08
x_{Orb}		-0.19	1.72	0.79	0.53	1.18	1.57
x_{Volc}			-0.15	-0.14	0.65	0.68	0.70
x_{Land}				0.61	0.36	0.54	-0.30
x_{Ghg}					0.38	0.46	0.96
x_{comb}						0.55	1.04
τ^+_{pseudo}							0.81

- *Summary of the results based on all 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\mathrm{Ssim}}$	0.064	0.078	0.086	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Volc}}\,\tilde{\delta}_{\mathsf{Land}}}$	0.014	0.015	0.016
$\widehat{\mathrm{Osim}}$	0.081	0.092	0.107	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Orb}}\,\tilde{\delta}_{\mathsf{Sol}}}$	0.007	0.007	0.008
$\widehat{\mathrm{Vsim}}$	0.196	0.201	0.204	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Orb}}\,\tilde{\delta}_{\mathsf{comb}}}$	-0.002	0.003	0.006
$\widehat{\mathrm{SG+}}$	0.200	0.239	0.270	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Land}}\,\tilde{\delta}_{\mathsf{Ghg}}}$	-0.0074	-0.0052	-0.0029
$\widehat{\mathrm{LG+}}$	0.230	0.340	0.420	$\widehat{\mathrm{Var}}(\xi^S_{\mathsf{Ghg\,(anthr)}})$	0.008	0.009	0.012
$\widehat{\mathrm{CG+}}$	-0.154	-0.110	-0.059	$\widehat{\sigma}^2_\nu$	0.06	0.083	0.101
Model χ^2	10.6	17.1	21.6	GFI	0.94	0.95	0.97
p-value	0.16	0.41	0.83	AGFI	0.90	0.92	0.95
SRMR	0.082	0.093	0.107				

The solution for each data set was proper.

6.3 North America (annual mean temperature)

Table S6.3.1. *The results of fitting the FA(2,1)-model and calculating the U_R- and U_T-test statistics. The results for the FA(2,1)-model and the U_R-statistic are based on the data sets $\{\bar{x}_{\text{single forcing}}, \bar{x}^{(i)}_{comb}\}$, where $\bar{x}^{(i)}_{comb}$ belongs to Data set number i, $i = 1, 2, \ldots, 10$, each of which is described in Table 1. The results for the U_T-statistic are based on the data sets $\{x_{\text{single forcing,repl. 1}}, \ldots, x_{\text{single forcing,repl. k}}, x_u, \bar{x}^{(i)}_{comb}\}$, where x_u is the unforced climate model simulation*

- Single forcing: *Solar*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.0498	< 0.01	0.0232	< 0.01	0.16	0.69	0.99	0.99	0.0241	2.69	-0.511

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.0473	0.023	0.001	0.31	0.990	0.969	0.002	1.90	-1.036
Mean	0.0494	0.024	0.242	0.72	0.998	0.993	0.023	2.54	-0.523
Max	0.0520	0.027	1.034	0.97	1.000	1.000	0.060	3.31	-0.088

- Single forcing: *Orbital*, $i = 10$. The independent estimate of $\sigma^{2\,*}_{\delta_{\text{Orb}}}$ turned out to be larger than the sample variance of $\bar{x}_{\text{Orb}}$ ($0.0071 > 0.0068$). To make the estimation meaningful, $\sigma^{2\,*}_{\delta_{\text{Orb}}}$ has been set to the sample variance of $\bar{x}_{\text{Orb}}$, i.e. 0.0068. This implies that $\alpha = 0$. Under this restriction the FA(2,1)-model has two degrees of freedom.

α	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0	0.026	< 0.01	1.112	0.57	0.99	0.98	0.061	1.03	0.20

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$

	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.0256	0.214	0.51	0.987	0.980	0.0268	0.452	0.028
Mean	0.0271	0.819	0.67	0.992	0.988	0.0512	0.863	0.215
Max	0.0307	1.349	0.90	0.998	0.997	0.0672	1.133	0.537

- Single forcing: *Volcanic*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.128	< 0.01	0.0089	< 0.01	0.16	0.69	0.99	0.99	0.020	17.43	-3.078

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$

	$\hat{a}_{11}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.127	0.0085	0.0013	0.407	0.993	0.979	0.0019	16.7	-3.42
Mean	0.128	0.0093	0.2743	0.661	0.997	0.992	0.0235	17.2	-3.13
Max	0.129	0.0125	0.6884	0.971	1.000	1.000	0.0419	17.7	-2.86

- Single forcing: *Land*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.066	< 0.01	0.022	< 0.01	0.004	0.95	0.99	0.99	0.0036	3.32	-0.38

Summary of the results for all data sets, $i = 1, 2, \ldots, 10$

	$\hat{a}_{11}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.064	0.022	0.0003	0.77	0.99	0.99	0.001	2.83	-0.56
Mean	0.065	0.023	0.0221	0.90	1.00	0.99	0.007	3.17	-0.39
Max	0.066	0.027	0.0895	0.99	1.00	1.00	0.018	3.40	-0.14

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- Single forcing: *Greenhouse gases (Ghg)*, $i = 10$

$\widehat{\alpha}$	p-value	$\widehat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.056	< 0.01	0.0221	< 0.01	0.89	0.35	0.99	0.97	0.057	3.30	-0.839

Summary of the results for all data sets, $i = 1, 2, \ldots, 10$

	$\widehat{\alpha}$	$\widehat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.054	0.022	0.43	0.30	0.989	0.97	0.039	2.86	-1.095
Mean	0.055	0.023	0.70	0.42	0.993	0.98	0.050	3.10	-0.821
Max	0.056	0.027	1.08	0.51	0.996	0.99	0.063	3.36	-0.448

Table S6.3.2. *The final factor model obtained by simplifying the basic FA(6,6)-factor model defined in Table 2 (North America)*

- *The result for Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	-0.048	1.2e-06	Vsim	0.126	3.3e-34
Lsim	-0.066	7.5e-09	Gsim	-0.051	2.3e-05
Isim	0.097	9.1e-06	ϕ_{LG}	0.861	1.3e-03
ϕ_{SI}	0.340	8.3e-02	ϕ_{LI}	0.823	1.5e-08
ϕ_{GI}	0.578	7.1e-03			

To assess the overall model fit

Model $\chi^2 = 16.42$, df $= 12$, p-value $= 0.1726$

$GFI = 0.94$, $AGFI = 0.90$, $SRMR = 0.111$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x_{comb}
x_{Sol}	0.00	0.19	1.77	2.60	1.2	1.67
x_{Orb}		0.00	0.71	-0.58	1.3	1.08
x_{Volc}			0.07	1.7	1.3	0.31
x_{Land}				0.00	0.00	1.40
x_{Ghg}					0.00	1.03
x_{comb}						0.38

- *Summary of the results based on all 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\mathrm{Ssim}}$	-0.048	-0.048	-0.048	$\widehat{\mathrm{Vsim}}$	0.125	0.127	0.128
$\widehat{\mathrm{Lsim}}$	-0.066	-0.066	-0.066	$\widehat{\mathrm{Gsim}}$	-0.051	-0.051	-0.051
$\widehat{\mathrm{Isim}}$	0.096	0.102	0.108	$\widehat{\phi}_{\mathrm{LG}}$	0.86	0.86	0.86
$\widehat{\phi}_{\mathrm{SI}}$	0.08	0.27	0.49	$\widehat{\phi}_{\mathrm{LI}}$	0.70	0.81	0.91
$\widehat{\phi}_{\mathrm{GI}}$	0.50	0.57	0.64				
Model χ^2	15.9	16.3	16.9	GFI	0.94	0.95	0.95
p-value	0.152	0.178	0.194	AGFI	0.90	0.90	0.91
SRMR	0.108	0.111	0.113				

The solution for each data set is proper

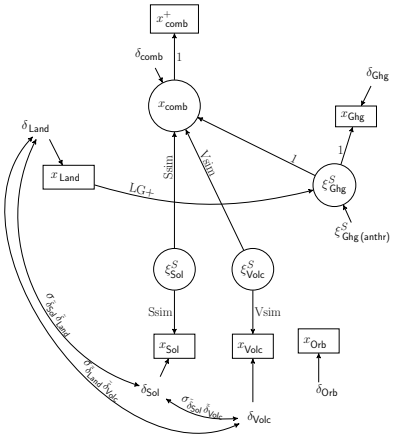

Figure S6.3.1. *Path diagram of the final SEM model fitted to the* North America *data in Step 1.*

Table S6.3.3. *The final SEM model for the North America data, depicted in Figure S6.3.1*

- *The result for Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.048	7.6e-10	$\sigma_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{Volc}}}$	2.1e-03	0.03
Vsim	0.126	3.8e-36	$\sigma_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{Land}}}$	2.2e-03	0.01
$\sigma^2_{\delta_{\text{Land}}}$	0.011	1.9e-12	$\sigma_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{Volc}}}$	2.5e-03	0.08
$\text{Var}(\xi^{S}_{\text{Ghg (anthr)}})$	2.0e-03	0.011			
LG+	0.296	2.1e-03			

To assess the overall model fit

Model $\chi^2 = 5.45$, df $= 13$, p-value $= 0.96$,
$GFI = 0.98$, $AGFI = 0.97$, $SRMR = 0.056$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^{+}_{comb}
x_{Sol}	0.00	-0.19	-0.07	0.15	0.44	-0.03
x_{Orb}		0.00	0.71	-0.58	1.22	1.07
x_{Volc}			0.07	0.60	0.95	0.13
x_{Land}				0.01	-0.25	0.67
x_{Ghg}					-0.32	0.76
x^{+}_{comb}						0.30

to be continued on the next page

• *Summary of the results based on all 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\text{Ssim}}$	0.045	0.051	0.057	$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{Volc}}}$	1.8e-03	2.2e-03	2.5e-03
$\widehat{\text{Vsim}}$	0.126	0.127	0.128	$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{Land}}}$	1.9e-03	2.1e-03	2.3e-03
$\widehat{\sigma}^2_{\delta_{\text{Land}}}$	0.0106	0.0106	0.0107	$\widehat{\sigma}_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{Volc}}}$	1.5e-03	1.7e-03	1.8e-03
$\widehat{\text{Var}}(\xi^S_{\text{Ghg (anthr)}})$	1.8e-03	2.1e-03	2.5e-03				
$\widehat{\text{LG+}}$	0.290	0.294	0.300				
Model χ^2	5.4	6.3	8.7	GFI	0.97	0.98	0.98
p-value	0.79	0.93	0.96	AGFI	0.95	0.97	0.97
SRMR	0.054	0.059	0.070				

The solution for each data set is proper

Under the SEM model in Figure S6.3.1, $\text{Var}(\xi^S_{\text{Ghg}}) = \sigma^2_{\text{Land}}\cdot(\text{LG+})^2 + \text{Var}(\xi^S_{\text{Ghg (anthr)}})$. For Data set number 10, $\widehat{\text{Var}}(\xi^S_{\text{Ghg}}) = 0.0029$ with the associated p-value $= 0.002$. Hence, the standardised coefficient for ξ^S_{Ghg} is estimated to be as large as $\sqrt{\widehat{\text{Var}}(\xi^S_{\text{Ghg}})} = \widehat{\text{Gsim}} = 0.054$, which, based on p-value $= 0.002$, is statistically significant at all conventional levels.

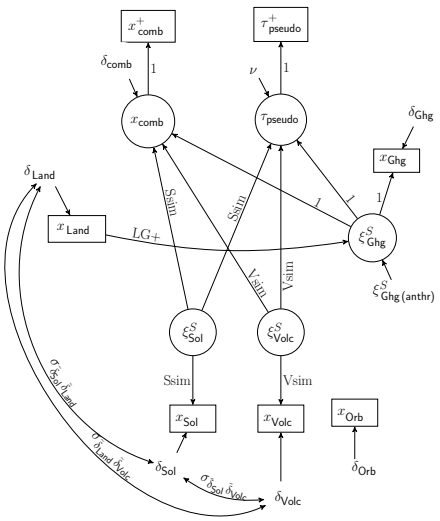

Figure S6.3.2. *Path diagram of the final SEM model fitted to the* North America *data in Step 2.*

Table S6.3.4. *The final SEM model for the North America data, depicted in Figure S6.3.2*

- *The result for Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.051	2.4e-11	$\sigma_{\tilde{\delta}_{\mathsf{Sol}}\,\tilde{\delta}_{\mathsf{Volc}}}$	2.3e-03	0.016
Vsim	0.126	1.2e-36	$\sigma_{\tilde{\delta}_{\mathsf{Sol}}\,\tilde{\delta}_{\mathsf{Land}}}$	2.1e-03	0.013
$\sigma^2_{\tilde{\delta}_{\mathsf{Land}}}$	0.011	1.9e-12	$\sigma_{\tilde{\delta}_{\mathsf{Land}}\,\tilde{\delta}_{\mathsf{Volc}}}$	1.6e-03	0.13
$\mathrm{Var}(\xi^S_{\mathsf{Ghg}\,(\mathsf{anthr})})$	2.1e-03	8.8e-03	σ^2_ν	0.016	1.0e-09
LG+	0.295	2.2e-04			

To assess the overall model fit

Model $\chi^2 = 9.72$ df $= 19$ p-value $= 0.96$

$GFI = 0.97$, $AGFI = 0.96$, $SRMR = 0.055$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}	τ^+_{pseudo}
x_{Sol}	-0.26	-0.19	-0.19	0.09	0.41	-0.22	0.16
x_{Orb}		0.00	0.71	-0.58	1.22	1.07	0.25
x_{Volc}			0.04	0.66	0.97	0.11	-0.08
x_{Land}				0.00	-0.24	0.67	0.584
x_{Ghg}					-0.35	0.72	0.30
x^+_{comb}						0.19	0.66
τ^+_{pseudo}							0.49

- *Summary of the results based on all 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\mathrm{Ssim}}$	0.046	0.051	0.057	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Sol}}\,\tilde{\delta}_{\mathsf{Volc}}}$	1.9e-03	2.3e-03	2.7e-03
$\widehat{\mathrm{Vsim}}$	0.126	0.127	0.128	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Sol}}\,\tilde{\delta}_{\mathsf{Land}}}$	2.0e-03	2.1e-03	2.3e-03
$\widehat{\sigma}^2_{\tilde{\delta}_{\mathsf{Land}}}$	0.0106	0.0106	0.0107	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Land}}\,\tilde{\delta}_{\mathsf{Volc}}}$	1.5e-03	1.6e-03	1.9e-03
$\widehat{\mathrm{Var}}(\xi^S_{\mathsf{Ghg}\,(\mathsf{anthr})})$	1.9e-03	2.1e-03	2.4e-03	$\widehat{\sigma}^2_\nu$	0.016	0.019	0.023
$\widehat{\mathrm{LG+}}$	0.290	0.294	0.302				
Model χ^2	6.9	11.5	18.9	GFI	0.95	0.97	0.98
p-value	0.46	0.87	0.99	AGFI	0.93	0.95	0.97
SRMR	0.055	0.063	0.077				

The solution for each data set is proper

6.4 Asia (summer, June-August, temperature)

Table S6.4.1. *The results of fitting the FA(2,1)-model and calculating the U_R- and U_T-test statistics. The results for the FA(2,1)-model and the U_R-statistic are based on the data sets $\{\bar{x}_{\text{single forcing}}, \bar{x}^{(i)}_{comb}\}$, where $\bar{x}^{(i)}_{comb}$ belongs to Data set number i, $i = 1, 2, \ldots, 10$, each of which is described in Table 1. The results for the U_T-statistic are based on the data sets $\{x_{\text{single forcing,repl. 1}}, \ldots, x_{\text{single forcing,repl. k}}, x_u, \bar{x}^{(i)}_{comb}\}$, where x_u is the unforced climate model simulation*

- Single forcing: *Solar*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.0254	< 0.01	0.0198	< 0.01	7.397	0.007	0.93	0.80	0.160	3.78	1.06

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.0242	0.019	5.96	0.002	0.911	0.734	0.144	3.24	0.752
Mean	0.0251	0.020	8.17	0.005	0.926	0.777	0.167	3.68	0.947
Max	0.0259	0.021	9.97	0.015	0.944	0.833	0.184	4.00	1.190

- Single forcing: *Orbital*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.031	< 0.01	0.020	< 0.01	0.009	0.92	0.99	0.99	0.006	1.548	1.82

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.0310	0.0200	0.000	0.733	0.99	0.997	0.000	1.42	1.38
Mean	0.0313	0.0206	0.034	0.880	1.00	0.999	0.009	1.60	1.64
Max	0.0316	0.0213	0.116	0.994	1.00	1.000	0.020	1.84	1.82

- Single forcing: *Volcanic*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.120	< 0.01	0.005	< 0.01	0.92	0.34	0.99	0.97	0.046	29.99	-3.77

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.120	0.005	0.486	0.131	0.98	0.93	0.036	29.0	-4.22
Mean	0.120	0.006	1.044	0.336	0.99	0.97	0.048	29.9	-3.94
Max	0.121	0.007	2.283	0.486	1.00	0.99	0.068	30.6	-3.48

- Single forcing: *Land*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.059	< 0.01	0.018	< 0.01	0.011	0.92	0.99	0.99	0.01	5.15	0.80

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.0587	0.017	0.00	0.64	0.99	0.99	0.00	4.55	0.26
Mean	0.0591	0.018	0.05	0.86	1.00	0.99	0.01	5.24	0.59
Max	0.0593	0.019	0.22	0.99	1.00	1.00	0.03	5.69	0.95

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- Single forcing: *Greenhouse gases (Ghg)*, $i = 10$

$\widehat{\alpha}$	p-value	$\widehat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.046	< 0.01	0.019	< 0.01	0.51	0.48	0.99	0.98	0.04	4.07	0.82

Summary of the results for all 10 data sets, $i = 1, 2, \ldots, 10$

	$\widehat{\alpha}$	$\widehat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.0456	0.0183	0.509	0.383	0.992	0.997	0.042	4.07	0.27
Mean	0.0457	0.0189	0.630	0.429	0.994	0.981	0.046	4.16	0.61
Max	0.0459	0.0195	0.760	0.475	0.995	0.985	0.051	4.26	0.97

Table S6.4.2. *The final factor model obtained by simplifying the basic FA(6,6)-factor model from Table 2 (Asia)*

- *The result for Data set number 10*

Parameter	Estimate	p-value
Ssim	0.024	1.3e-05
Osim	0.018	1.0e-02
Vsim	0.118	6.1e-40
Lsim	0.048	3.8e-18
Gsim	0.041	1.2e-14
ϕ_{LG}	-0.109	5.2e-01

To assess the overall model fit

Model $\chi^2 = 34.01$ df $= 15$ p-value $= 0.003$

$GFI = 0.89$, $AGFI = 0.85$, $SRMR = 0.1462$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x_{comb}
x_{Sol}	-0.75	1.18	1.39	3.24	0.67	2.59
x_{Orb}		1.04	0.41	3.00	1.52	1.04
x_{Volc}			0.07	1.26	1.49	0.50
x_{Land}				0.58	0.80	1.38
x_{Ghg}					0.55	1.34
x_{comb}						0.66

- *Summary of the results for all 10 data sets*

	Min	Mean	Max
$\widehat{\mathrm{Ssim}}$	0.0210	0.0235	0.0255
$\widehat{\mathrm{Osim}}$	0.0158	0.0186	0.0208
$\widehat{\mathrm{Vsim}}$	0.1170	0.0118	0.119
$\widehat{\mathrm{Lsim}}$	0.0460	0.0480	0.0490
$\widehat{\mathrm{Gsim}}$	0.0400	0.0410	0.0430
$\widehat{\phi}_{LG}$	-0.132	-0.087	-0.059
Model χ^2	31	33	36
GFI	0.89	0.89	0.90
AGFI	0.84	0.85	0.86
SRMR	0.142	0.145	0.146

The solution for each data set is proper.

Table S6.4.3. *The result of estimating the TLS- FA(6,5)- model from Table 3 (Asia)*

• *The result for Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.0240	1.3e-05	ϕ_{LG}	-0.3418	4.5e-01
$\text{Strue}_{\text{pseudo}}$	0.066	7.8e-01	ϕ_{SO}	0.4817	2.5e-01
Gsim	0.0444	9.8e-13	ϕ_{SV}	0.3200	1.3e-01
$\text{Gtrue}_{\text{pseudo}}$	0.0228	8.6e-01	ϕ_{SL}	0.7409	1.1e-01
Osim	0.0261	5.5e-04	ϕ_{SG}	0.2279	4.4e-01
$\text{Otrue}_{\text{pseudo}}$	0.0016	9.9e-01	ϕ_{OV}	0.0796	7.1e-01
Lsim	0.0356	1.8e-02	ϕ_{OL}	1.0443	1.9e-01
$\text{Ltrue}_{\text{pseudo}}$	-0.0102	9.7e-01	ϕ_{OG}	0.3903	1.7e-01
Vsim	0.1187	1.3e-38	ϕ_{VL}	0.1580	5.3e-01
$\text{Vtrue}_{\text{pseudo}}$	0.1014	9.3e-07	ϕ_{VG}	0.2111	1.4e-01

To assess the overall model fit

Model $\chi^2 = 1.38$ df $= 1$ p-value $= 0.24$

$GFI = 1$, $AGFI = 0.91$, $SRMR = 0.025$

• *Summary of the results for all 10 data sets*

The estimation procedure failed to converge to a solution for Data sets number 1, 4 and 5. For the remaining data sets, the solutions contain $\widehat{\phi}_{OL}$ exceeding 1.

Table S6.4.4. *The 95% Fieller confidence region (CR) for the ratios of the factor loadings presented in Table S6.4.3*

Ratio	Estimate	The associated 95% Fieller CR
$\text{Strue}_{\text{pseudo}}$/Ssim	3.17	$(-18.03, 28.38)$
$\text{Gtrue}_{\text{pseudo}}$/Gsim	0.51	$(-5.19, 6.42)$
$\text{Otrue}_{\text{pseudo}}$/Osim	0.06	$(-19.06, 19.63)$
$\text{Ltrue}_{\text{pseudo}}$/Lsim	-0.29	$(-27.21, 21.63)$
$\text{Vtrue}_{\text{pseudo}}$/Vsim	0.85	$(0.52, 1.19)$

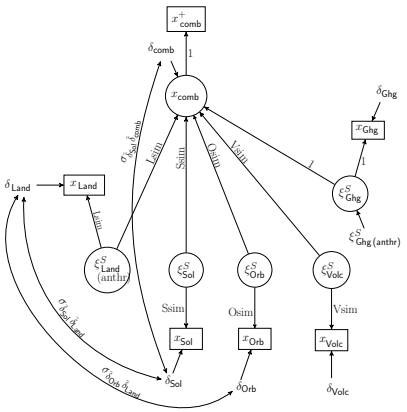

Figure S6.4.1. *Path diagram of the first SEM model fitted to the* Asia *data in Step 1 under the assumption that the effect of natural changes in vegetation, associated with the vegetation phenology, is negligible.*

Table S6.4.5. *The result of estimating the SEM model presented graphically in Figure S6.4.1.*

- *The result based on Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.017	0.045	$\sigma_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{Land}}}$	9.2e-04	2.2e-03
Osim	0.022	1.8e-04	$\sigma_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Land}}}$	7.9e-04	0.013
Vsim	0.118	5.1e-40	$\sigma_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{comb}}}$	0.0012	8.9e-05
Lsim	0.049	6.1e-21	$\text{Var}(\xi^S_{\text{Ghg (anthr)}})$	0.0016	8.1e-05

To assess the overall model fit
Model $\chi^2 = 10.3$ df $= 13$ p-value $= 0.67$
$GFI = 0.96$ $AGFI = 0.94$ $SRMR = 0.089$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}
x_{Sol}	0.13	1.22	1.49	0.37	0.72	1.30
x_{Orb}		0.50	0.39	0.51	1.48	0.75
x_{Volc}			0.11	1.25	1.51	0.54
x_{Land}				0.42	0.26	1.04
x_{Ghg}					0.70	1.17
x^+_{comb}						0.56

- *Summary of the results based on the 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\text{Ssim}}$	0.016	0.017	0.019	$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{Land}}}$	3.6e-04	8.7e-04	1.0e-03
$\widehat{\text{Osim}}$	0.020	0.023	0.026	$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Land}}}$	7.6e-04	9.2e-04	1.0e-03
$\widehat{\text{Vsim}}$	0.117	0.118	0.119	$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{comb}}}$	9.5e-04	1.1e-03	1.3e-03
$\widehat{\text{Lsim}}$	0.047	0.050	0.051	$\widehat{\text{Var}}(\xi^S_{\text{Ghg (anthr)}})$	0.0016	0.0016	0.0017
Model χ^2	9.1	9.7	12.7	GFI	0.96	0.97	0.97
p-value	0.52	0.66	0.76	AGFI	0.94	0.94	0.95
SRMR	0.085	0.090	0.096				

Under the SEM model in Figure S6.4.1, $\text{Var}(\xi^S_{\text{Ghg}}) = \text{Var}(\xi^S_{\text{Ghg (anthr)}})$. For Data set number 10, $\widehat{\text{Var}}(\xi^S_{\text{Ghg}}) = 0.0016$ with the associated p-value=8.1e-05. Hence, the standardised coefficient for ξ^S_{Ghg} is estimated to be as large as $\sqrt{\widehat{\text{Var}}(\xi^S_{\text{Ghg}})} = 0.040$, which, based on p-value=8.1e-05, is statistically significant at all conventional levels.

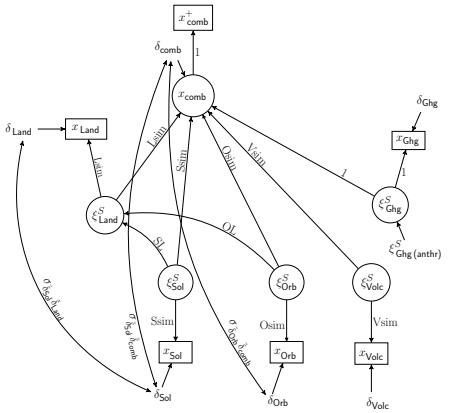

Figure S6.4.2. *Path diagram of the second SEM model fitted to the* Asia *data in Step 1 without assuming that the effect of natural changes in vegetation, associated with the vegetation phenology, is negligible.*

Table S6.4.6. *The result of estimating the SEM model presented graphically in Figure S6.4.2.*

- *The result based on Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.019	6.5e-03	$\sigma_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{Land}}}$	1.8e-03	1.3e-03
Osim	0.023	1.8e-04	$\sigma_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{comb}}}$	1.9e-03	1.9e-06
Vsim	0.118	5.1e-40	$\sigma_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{comb}}}$	-8.9e-04	0.076
OL	0.028	0.016	$\text{Var}(\xi^S_{\text{Ghg (anthr)}})$	0.0016	8.1e-05
SL	-0.041	8.2e-05			

To assess the overall model fit

Model $\chi^2 = 10.13$ df $= 12$ p-value $= 0.60$

$GFI = 0.96$ $AGFI = 0.94$ $SRMR = 0.090$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}
x_{Sol}	-0.16	1.18	1.46	0.16	0.71	1.16
x_{Orb}		0.30	0.39	0.86	1.46	0.97
x_{Volc}			0.10	1.24	1.51	0.53
x_{Land}				0.27	0.26	1.06
x_{Ghg}					0.68	1.16
x^+_{comb}						0.55

to be continued on the next page

- *Summary of the results based on the 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\text{Ssim}}$	0.017	0.019	0.021	$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{Land}}}$	1.6e-03	1.8e-03	1.9e-03
$\widehat{\text{Osim}}$	0.021	0.024	0.027	$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{comb}}}$	1.8e-03	1.9e-03	2.1e-03
$\widehat{\text{Vsim}}$	0.117	0.118	0.119	$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{comb}}}$	-1.1e-03	-8.8e-04	-5.5e-04
$\widehat{\text{OL}}$	0.024	0.029	0.031	$\widehat{\text{Var}}(\xi^S_{\text{Ghg (anthr)}})$	0.0016	0.0017	0.0018
$\widehat{\text{SL}}$	-0.040	-0.041	-0.043				
Model χ^2	9.1	10.2	12.2	GFI	0.96	0.96	0.97
p-value	0.43	0.60	0.70	AGFI	0.93	0.94	0.94
SRMR	0.086	0.090	0.095				

Under the SEM model in Figure S6.4.2, $\text{Var}(\xi^S_{\text{Land}}) = \text{SL}^2 + \text{OL}^2$. For Data set number 10, $\widehat{\text{Var}}(\xi^S_{\text{Land}}) = 0.0025$ (p-value = 4.1e-05). Hence, the stand. coefficient for ξ^S_{Land} is estimated to be as large as $\widehat{\text{Lsim}} = \sqrt{\widehat{\text{Var}}(\xi^S_{\text{Land}})} = 0.050$, which is statistically significant at all conventional significance levels. As for $\widehat{\text{Var}}(\xi^S_{\text{Ghg}})$, its expression and the associated conclusions, based on Data set number 10, remain the same as under the SEM model in Figure S6.4.1.

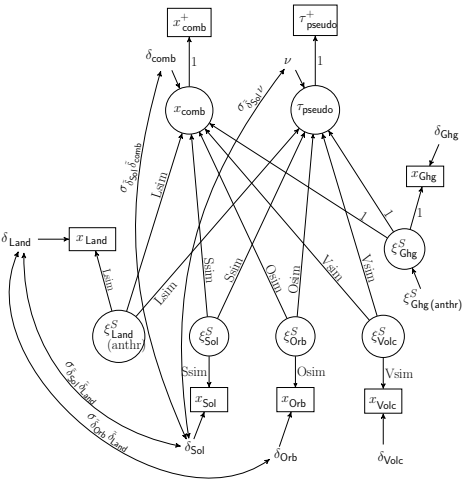

Figure S6.4.3. *Path diagram of the first SEM model fitted to the Asia data in Step 2 under the assumption that the effect of natural changes in vegetation, associated with the vegetation phenology, is negligible.*

Table S6.4.7 *The result of fitting the SEM model depicted in Figure S6.4.3*

- *The result for Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.017	0.048	$\sigma_{\tilde{\delta}_{\mathsf{Sol}}\,\tilde{\delta}_{\mathsf{Land}}}$	9.1e-04	2.5e-03
Osim	0.021	4.7e-04	$\sigma_{\tilde{\delta}_{\mathsf{Orb}}\,\tilde{\delta}_{\mathsf{Land}}}$	8.0e-04	0.012
Vsim	0.117	4.3e-40	$\sigma_{\tilde{\delta}_{\mathsf{Sol}}\,\tilde{\delta}_{\mathsf{comb}}}$	0.0012	7.2e-05
Lsim	0.049	1.6e-21	$\sigma_{\tilde{\delta}_{\mathsf{Sol}}\,\nu}$	0.0013	3.0e-03
$\mathrm{Var}(\xi^S_{\mathsf{Ghg\,(anthr)}})$	0.0016	6.5e-05	σ^2_ν	0.0061	7.8e-10

To assess the overall model fit

Model $\chi^2 = 13.29$ df $= 18$ p-value $= 0.77$

$GFI = 0.96$ $AGFI = 0.94$ $SRMR = 0.091$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}	τ^+_{pseudo}
x_{Sol}	0.15	1.23	1.49	0.40	0.72	1.30	1.31
x_{Orb}		0.64	0.40	0.49	1.50	0.83	0.15
x_{Volc}			0.13	1.25	1.51	0.56	0.43
x_{Land}				0.35	0.26	0.98	1.32
x_{Ghg}					0.67	1.16	1.28
x^+_{comb}						0.57	0.55
τ^+_{pseudo}							0.49

- *Summary of the results for all 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\mathrm{Ssim}}$	0.015	0.017	0.018	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Sol}}\,\tilde{\delta}_{\mathsf{Land}}}$	8.4e-04	9.1e-04	9.6e-03
$\widehat{\mathrm{Osim}}$	0.021	0.023	0.025	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Orb}}\,\tilde{\delta}_{\mathsf{Land}}}$	7.6e-04	8.0e-04	8.5e-03
$\widehat{\mathrm{Vsim}}$	0.117	0.118	0.118	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Sol}}\,\tilde{\delta}_{\mathsf{comb}}}$	9.4e-04	1.1e-03	1.3e-03
$\widehat{\mathrm{Lsim}}$	0.048	0.049	0.051	$\widehat{\sigma}_{\tilde{\delta}_{\mathsf{Sol}}\,\nu}$	8.7e-04	1.1e-03	1.7e-03
$\widehat{\mathrm{Var}}(\xi^S_{\mathsf{Ghg\,(anthr)}})$	0.0016	0.0017	0.0017	$\widehat{\sigma}^2_\nu$	0.0045	0.0060	0.0075
Model χ^2	9.85	15.6	22.2	GFI	0.94	0.96	0.97
p-value	0.23	0.62	0.94	AGFI	0.91	0.93	0.95
SRMR	0.088	0.091	0.094				

The solution for each data set is proper

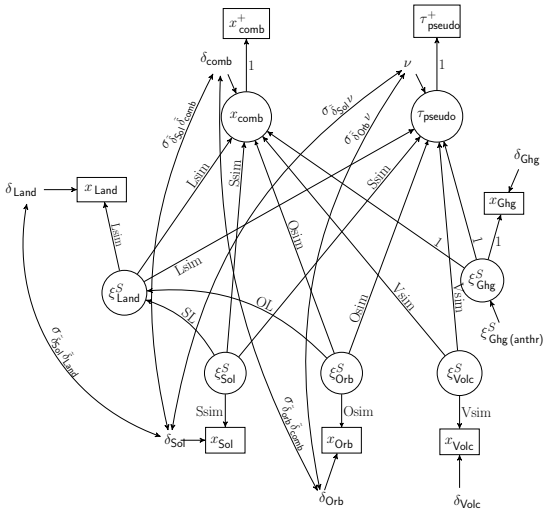

Figure S6.4.4. *Path diagram of the second SEM model fitted to the Asia data in Step 2 without assuming that the effect of natural changes in vegetation, associated with the vegetation phenology, is negligible.*

Table S6.4.8 *The result of fitting the SEM model depicted in Figure S6.4.4*

- *The result for Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.019	5.4e-03	$\sigma_{\tilde{\delta}_{\text{Sol}}\tilde{\delta}_{\text{Land}}}$	1.8e-03	1.1e-03
Osim	0.026	1.3e-04	$\sigma_{\tilde{\delta}_{\text{Sol}}\tilde{\delta}_{\text{comb}}}$	2.0e-04	1.9e-06
Vsim	0.118	3.7e-40	$\sigma_{\tilde{\delta}_{\text{Sol}}\nu}$	2.2e-03	3.0e-05
OL	0.029	6.3e-03	$\sigma_{\tilde{\delta}_{\text{Orb}}\tilde{\delta}_{\text{comb}}}$	-1.1e-03	0.036
SL	-0.041	5.6e-05	$\sigma_{\tilde{\delta}_{\text{Orb}}\nu}$	-1.6e-03	0.016
$\text{Var}(\xi^S_{\text{Ghg (anthr)}})$	0.0017	5.5e-05	σ^2_ν	0.0066	8.5e-11

To assess the overall model fit

Model $\chi^2 = 12.06$ df $= 16$ p-value $= 0.74$

$GFI = 0.96$ $AGFI = 0.94$ $SRMR = 0.089$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}	τ^+_{pseudo}
x_{Sol}	-0.19	1.16	1.45	0.17	0.70	1.19	0.94
x_{Orb}		0.07	0.38	0.65	1.43	0.96	0.97
x_{Volc}			0.10	1.24	1.50	0.53	0.40
x_{Land}				0.25	0.26	0.98	1.29
x_{Ghg}					0.62	1.13	1.23
x^+_{comb}						0.64	0.44
τ^+_{pseudo}							0.26

to be continued on the next page

- *Summary of the results for all 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\text{Ssim}}$	0.011	0.016	0.021	$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{Land}}}$	4.5e-04	1.7e-03	1.9e-03
$\widehat{\text{Osim}}$	0.019	0.025	0.028	$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{comb}}}$	9.6e-04	1.8e-03	2.0e-03
$\widehat{\text{Vsim}}$	0.117	0.118	0.118	$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\nu}$	7.4e-04	1.9e-03	2.5e-03
$\widehat{\text{OL}}$	0.023	0.028	0.030	$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{comb}}}$	-1.1e-03	-9.3e-04	-4.7e-04
$\widehat{\text{SL}}$	-0.036	-0.041	-0.043	$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\nu}$	-1.6e-03	-9.4e-04	-5.5e-04
$\widehat{\text{Var}}(\xi^S_{\text{Ghg (anthr)}})$	0.0016	0.0017	0.0017	$\widehat{\sigma}^2_\nu$	0.0049	0.0064	0.0077
Model χ^2	10.7	15.4	20.3	GFI	0.94	0.96	0.97
p-value	0.21	0.50	0.83	AGFI	0.90	0.92	0.95
SRMR	0.081	0.089	0.101				

The solution for each data set is proper

6.5 South America (warm-season (Dec-Feb) mean temperature)

Table S6.5.1. *The results of fitting the FA(2,1)-model and calculating the U_R- and U_T-test statistics. The results for the FA(2,1)-model and the U_R-statistic are based on the data sets $\{\bar{x}_{\text{single forcing}}, \bar{x}^{(i)}_{\text{comb}}\}$, where $\bar{x}^{(i)}_{\text{comb}}$ belongs to Data set number i, $i = 1, 2, \ldots, 10$, each of which is described in Table 1. The results for the U_T-statistic are based on the data sets $\{x_{\text{single forcing,repl. 1}}, \ldots, x_{\text{single forcing,repl. k}}, x_u, \bar{x}^{(i)}_{\text{comb}}\}$, where x_u is the unforced climate model simulation*

- Single forcing: *Solar*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.032	< 0.01	0.0148	< 0.01	3.69	0.055	0.96	0.89	0.127	2.60	-1.58

Summary of the results for all 10 data sets

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.029	0.0133	2.58	0.015	0.94	0.83	0.105	2.19	-2.07
Mean	0.032	0.0148	4.21	0.047	0.96	0.88	0.135	2.73	-1.74
Max	0.035	0.0168	5.96	0.108	0.98	0.92	0.162	3.23	-1.38

- Single forcing: *Orbital*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.039	< 0.01	0.0151	< 0.01	0.781	0.377	0.99	0.98	0.056	0.81	-0.50

Summary of the results for all 10 data sets

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.036	0.0139	0.123	0.25	0.99	0.96	0.022	0.49	-0.91
Mean	0.039	0.0151	0.624	0.46	0.99	0.98	0.048	0.98	-0.68
Max	0.043	0.0166	1.333	0.73	1.00	0.99	0.073	1.60	-0.36

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- Single forcing: *Volcanic*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.104	$<< 0.01$	0.005	< 0.01	0.52	0.47	0.99	0.98	0.035	15.18	-3.85

Summary of the results for all 10 data sets

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.102	0.004	0.001	0.47	0.99	0.98	0.001	13.8	-4.11
Mean	0.103	0.005	0.263	0.67	0.99	0.99	0.022	14.7	-3.84
Max	0.104	0.007	0.523	0.98	1.00	1.00	0.037	15.5	-3.49

- Single forcing: *Land*, $i = 10$. The independent estimate of $\sigma^{2\,*}_{\delta_{\mathsf{Land}}}$ turned out to be larger than the sample variance of $\bar{x}_{\mathsf{Land}}$ ($0.0053 > 0.0049$). To make the estimation meaningful, $\sigma^{2\,*}_{\delta_{\mathsf{Land}}}$ is set to the sample variance of $\bar{x}_{\mathsf{Land}}$, i.e. 0.0049. This implies that $\alpha = 0$. Under this restriction the FA(2,1)-model has two degrees of freedom.

α	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0	0.015	$<< 0.01$	4.061	0.044	0.96	0.88	0.141	2.07	-2.07

Summary of the results for all 10 data sets

	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.014	2.33	0.012	0.94	0.82	0.106	1.48	-2.71
Mean	0.015	4.47	0.046	0.96	0.87	0.146	2.15	-2.33
Max	0.018	6.37	0.137	0.98	0.93	0.178	2.67	-1.90

- Single forcing: *Greenhouse gases (Ghg)*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.026	0.051	0.0156	$<< 0.01$	0.004	0.95	1	0.99	0.004	0.753	-1.23

Summary of the results for all 10 data sets

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.021	0.014	0.004	0.34	0.99	0.97	0.004	0.177	-1.82
Mean	0.027	0.016	0.172	0.74	0.99	0.99	0.022	0.955	-1.43
Max	0.034	0.018	0.897	0.95	1.00	1.00	0.063	1.949	-1.08

Table S6.5.2. *The final factor model obtained by simplifying the basic FA(6,6)-factor model from Table 2 (South America)*

- *The result for Data set number 10*

Parameter	Estimate	p-value
Ssim	0.020	0.085
Osim	0.039	3.8e-06
Vsim	0.104	3.1e-32
Gsim	0.020	0.130

To assess the overall model fit

Model $\chi^2 = 23.13$ df $= 17$ p-value $= 0.145$
$GFI = 0.93$, $AGFI = 0.92$, $SRMR = 0.101$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x_{comb}
x_{Sol}	-0.26	0.20	1.91	1.50	1.49	2.02
x_{Orb}		0.46	-1.35	-0.15	-0.31	-0.69
x_{Volc}			-0.23	0.32	0.01	0.17
x_{Land}				0.00	0.24	2.12
x_{Ghg}					0.30	0.33
x_{comb}						-0.15

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• *Summary of the results for all 10 data sets*

	Min	Mean	Max
$\widehat{\text{Ssim}}$	0.019	0.021	0.025
$\widehat{\text{Osim}}$	0.038	0.042	0.046
$\widehat{\text{Vsim}}$	0.101	0.103	0.105
$\widehat{\text{Gsim}}$	0.017	0.022	0.030
Model χ^2	16	24	35
p-value	0.01	0.17	0.54
GFI	0.91	0.93	0.95
AGFI	0.88	0.91	0.94
SRMR	0.092	0.102	0.115

The solution for each data set is proper

Table S6.5.3. *The result of estimating the TLS-FA(6,5)- model from Table 3 (South America)*

• *The result for Data set number 2*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.027	3.6e-03	ϕ_{SO}	0.15	0.78
$\text{Strue}_{\text{pseudo}}$	0.100	0.37	ϕ_{SV}	0.59	0.05
Osim	0.046	2.9e-04	ϕ_{SL}	6.99	0.97
$\text{Otrue}_{\text{pseudo}}$	0.015	0.87	ϕ_{SG}	0.97	0.23
Vsim	0.102	5.5e-04	ϕ_{OV}	-0.30	0.18
$\text{Vtrue}_{\text{pseudo}}$	0.048	0.68	ϕ_{OL}	-0.62	0.97
Lsim	0.004	0.97	ϕ_{OG}	-0.19	0.75
$\text{Ltrue}_{\text{pseudo}}$	-0.002	0.97	ϕ_{VL}	0.52	0.97
Gsim	0.026	0.12	ϕ_{VG}	-0.003	0.99
$\text{Gtrue}_{\text{pseudo}}$	-0.020	0.92	ϕ_{LG}	1.53	0.97

To assess the overall model fit

Model $\chi^2 = 0.93$ df $= 1$ p-value $= 0.33$

$GFI = 0.99$, $AGFI = 0.94$, $SRMR = 0.025$

For the remaining 9 data sets, the estimation algorithm failed to converge to a solution.

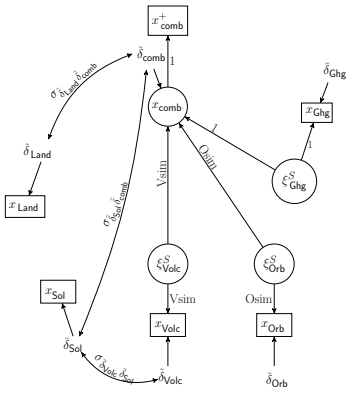

Figure S6.5.1. *Path diagram of the final SEM model fitted to the* South America *data in Step 1.*

Table S6.5.4. *The result of estimating the final SEM model depicted in Figure S6.5.1*

• *The result for Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Osim	0.047	1.6e-09	$\sigma_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{Sol}}}$	1.5e-03	0.051
Vsim	0.105	2.6e-33	$\sigma_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{comb}}}$	1.7e-03	2.8e-03
			$\sigma_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{comb}}}$	1.7e-03	0.034

To assess the overall model fit

Model $\chi^2 = 8.82$ df $= 16$ p-value$= 0.92$

$GFI = 0.97$, $AGFI = 0.96$, $SRMR = 0.069$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}
x_{Sol}	0.34	0.20	0.06	1.60	1.62	0.51
x_{Orb}		-0.08	-1.28	-0.14	-0.31	-0.91
x_{Volc}			-0.38	0.31	0.01	-0.01
x_{Land}				0.00	0.25	0.31
x_{Ghg}					0.83	0.29
x^+_{comb}						0.25

• *Summary of the results based on all 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\text{Osim}}$	0.041	0.045	0.049	$\widehat{\sigma}_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{Sol}}}$	1.4e-03	1.5e-03	1.6e-03
$\widehat{\text{Vsim}}$	0.101	0.104	0.105	$\widehat{\sigma}_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{comb}}}$	1.1e-03	1.7e-03	2.2e-03
				$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{comb}}}$	1.6e-03	1.9e-03	2.2e-03
Model χ^2	8.8	11.7	16.2	GFI	0.95	0.96	0.97
p-value	0.40	0.75	0.92	AGFI	0.93	0.95	0.96
SRMR	0.068	0.071	0.078				

The solution for each data set is proper

Under the SEM model depicted in Figure S6.5.1, $\text{Var}(\xi^S_{\text{Ghg}}) = 0$ because no causal path leads to ξ^S_{Ghg}. Hence, the estimate of the stand. coef. for ξ^S_{Ghg} is zero as well.

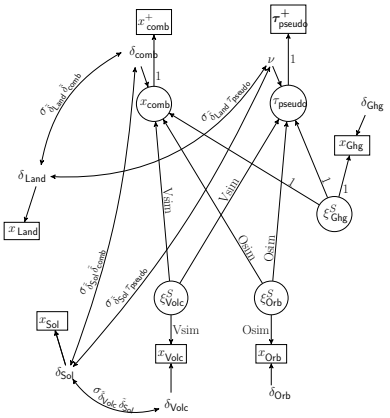

Figure S6.5.2. *Path diagram of the final SEM model fitted to the* South America *data in Step 2.*

Table S6.5.5. *The result of estimating the SEM model depicted in Figure S6.5.2*

• *The result for Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Osim	0.044	9,7e-09	$\sigma_{\tilde{\delta}_{\mathsf{Volc}}\,\tilde{\delta}_{\mathsf{Sol}}}$	1.5e-03	0.055
Vsim	0.105	7.6e-34	$\sigma_{\tilde{\delta}_{\mathsf{Land}}\,\tilde{\delta}_{\mathsf{comb}}}$	1.8e-03	6.1e-04
σ^2_ν	0.0183	1.4e-10	$\sigma_{\tilde{\delta}_{\mathsf{Land}}\,\nu}$	0.00	0.95
$\sigma_{\tilde{\delta}_{\mathsf{Sol}}\,\tilde{\delta}_{\mathsf{comb}}}$	1.8e-03	0.026	$\sigma_{\tilde{\delta}_{\mathsf{Sol}}\,\nu}$	1.5e-03	0.1

To assess the overall model fit

Model $\chi^2 = 12.0$ df $= 20$ p-value $= 0.92$

$GFI = 0.97$, $AGFI = 0.95$, $SRMR = 0.063$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}	τ^+_{pseudo}
x_{Sol}	0.34	0.21	0.09	1.60	1.62	0.49	0.12
x_{Orb}		0.17	-1.31	-0.14	-0.31	-0.94	0.06
x_{Volc}			-0.34	0.32	0.06	0.05	-0.15
x_{Land}				0.00	0.25	0.13	0.18
x_{Ghg}					0.83	0.75	0.92
x^+_{comb}						-0.04	0.17
τ^+_{pseudo}							0.13

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• Summary of the results based on all 10 data sets

	Min	Mean	Max		Min	Mean	Max
$\widehat{\text{Osim}}$	0.041	0.045	0.049	$\widehat{\sigma}_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{Sol}}}$	-3.5e-03	1.0e-03	1.7e-03
$\widehat{\text{Vsim}}$	0.101	0.104	0.105	$\widehat{\sigma}_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{comb}}}$	1.1e-03	2.2e-03	6.9e-03
$\widehat{\sigma}^2_{\nu}$	0.014	0.018	0.021	$\widehat{\sigma}_{\tilde{\delta}_{\text{Land}}\,\tau_{\text{pseudo}}}$	-1.0e.04	1.8e-03	4.3e-03
$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{comb}}}$	-4.3e-03	1.7e-03	2.2e-03	$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\nu}$	1.1e-03	2.8e-03	1.0e-02
Model χ^2	12.0	16.4	20.9	GFI	0.94	0.96	0.97
p-value	0.40	0.68	0.92	AGFI	0.94	0.96	0.97
SRMR	0.063	0.071	0.092				

The solution for each data set is proper

6.6 Australasia (warm-season (Sep-Feb) mean temperature)

Table S6.6.1. *The results of fitting the FA(2,1)-model and calculating the U_R- and U_T-test statistics. The results for the FA(2,1)-model and the U_R-statistic are based on the data sets $\{\bar{x}_{\text{single forcing}}, \bar{x}^{(i)}_{\text{comb}}\}$, where $\bar{x}^{(i)}_{\text{comb}}$ belongs to Data set number i, $i = 1, 2, \ldots, 10$, each of which is described in Table 1. The results for the U_T-statistic are based on the data sets $\{x_{\text{single forcing, repl.1}}, \ldots, x_{\text{single forcing, repl.}k}, x_u, \bar{x}^{(i)}_{\text{comb}}\}$, where x_u is the unforced climate model simulation*

• Single forcing: *Solar*, $i = 10$

$\widehat{\alpha}$	p-value	$\widehat{\sigma}^2_{\nu}$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.029	<< 0.01	0.0079	<< 0.01	2.36	0.12	0.98	0.93	0.089	4.39	1.01

Summary of the results for all 10 data sets

	$\widehat{\alpha}$	$\widehat{\sigma}^2_{\nu}$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.028	0.0071	1.24	0.059	0.97	0.89	0.065	3.93	0.63
Mean	0.029	0.0078	2.41	0.138	0.98	0.93	0.088	4.40	1.00
Max	0.030	0.0085	3.57	0.266	0.99	0.96	0.100	4.81	1.53

• Single forcing: *Orbital*, $i = 10$

$\widehat{\alpha}$	p-value	$\widehat{\sigma}^2_{\nu}$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.0191	< 0.01	0.0086	<< 0.01	2.28	0.13	0.98	0.93	0.092	2.42	1.23

Summary of the results for all 10 data sets

	$\widehat{\alpha}$	$\widehat{\sigma}^2_{\nu}$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.018	0.0076	1.03	0.058	0.97	0.90	0.062	1.82	0.72
Mean	0.019	0.0086	2.40	0.146	0.98	0.93	0.093	2.44	1.21
Max	0.020	0.0095	3.58	0.310	0.99	0.97	0.116	2.91	1.60

• Single forcing: *Volcanic*, $i = 10$

$\widehat{\alpha}$	p-value	$\widehat{\sigma}^2_{\nu}$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.078	<< 0.01	0.0028	<< 0.01	0.19	0.66	0.99	0.99	0.021	21.6	-1.63

Summary of the results for all 10 data sets

	$\widehat{\alpha}$	$\widehat{\sigma}^2_{\nu}$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.077	0.0024	0.017	0.18	0.98	0.95	0.007	21.2	-2.12
Mean	0.078	0.0027	0.531	0.56	0.99	0.98	0.030	21.8	-1.71
Max	0.079	0.0028	1.805	0.90	1.00	1.00	0.063	22.3	-1.25

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- Single forcing: *Land*, $i = 10$.

The independent estimate of $\sigma^{2\,*}_{\delta_{\mathsf{Land}}}$ turned out to be larger than the sample variance of $\bar{x}_{\mathsf{Land}}$ ($0.0015 > 0.0013$). To make the estimation meaningful, $\sigma^{2\,*}_{\delta_{\mathsf{Land}}}$ has been set to the sample variance of $\bar{x}_{\mathsf{Land}}$, i.e. 0.0013. This implies that $\alpha = 0$. Under this restriction the FA(2,1)-model has two degrees of freedom.

α	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0	0.0092	$<< 0.01$	8.14	0.02	0.93	0.89	0.162	2.67	1.08

Summary of the results for all 10 data sets

	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.0082	5.79	0.005	0.91	0.86	0.138	2.27	0.71
Mean	0.0092	8.18	0.021	0.93	0.89	0.162	2.67	1.07
Max	0.0099	10.5	0.055	0.95	0.92	0.183	3.01	1.42

- Single forcing: *Greenhouse gases (Ghg)*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.029	$<< 0.01$	0.0092	$<< 0.01$	6.1	0.014	0.94	0.83	0.15	-0.41	3.19

Summary of the results for all 10 data sets

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	-0.028	0.0082	5.51	0.004	0.93	0.78	0.14	-0.99	2.80
Mean	0.023	0.0092	6.77	0.010	0.94	0.81	0.16	-0.61	3.27
Max	0.029	0.0100	8.15	0.019	0.95	0.85	0.17	-0.16	3.66

Table S6.6.2. *The final FA(6,4)-factor model obtained by simplifying the basic FA(6,6)-model from Table 2 (Australasia)*

- *The result for Data set number 10*

Parameter	Estimate	p-value
Ssim	0.028	8.5e-14
Osim	0.020	6.6e-06
Vsim	0.078	2.1e-37
Gsim	0.028	1.3e-11

To assess the overall model fit
Model $\chi^2 = 26.12$ df $= 17$ p-value $= 0.072$
$GFI = 0.92$, $AGFI = 0.90$, $SRMR = 0.131$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x_{comb}
x_{Sol}	-0.30	0.72	1.16	1.30	0.94	1.32
x_{Orb}		-0.75	0.55	1.80	0.64	1.10
x_{Volc}			-0.20	1.57	-2.37	0.03
x_{Land}				0.00	0.82	2.83
x_{Ghg}					1.18	-2.10
x_{comb}						0.18

- Summary of the results based on all 10 data sets

	Min	Mean	Max		Min	Mean	Max
$\widehat{\text{Ssim}}$	0.026	0.028	0.030	$\widehat{\text{Osim}}$	0.018	0.020	0.021
$\widehat{\text{Vsim}}$	0.077	0.079	0.080	$\widehat{\text{Gsim}}$	0.026	0.027	0.029
Model χ^2	24.1	27.5	31.3	GFI	0.91	0.91	0.92
p-value	0.018	0.058	0.116	AGFI	0.89	0.89	0.90
SRMR	0.125	0.132	0.139				

The solution for each data set is proper

Table S6.6.3. *The result of estimating the TLS-FA(6,5)- model from Table 3 (Australasia)*

- *The result for Data set number 3*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.027	2.7e-09	ϕ_{SO}	0.31	0.41
$\text{Strue}_{\text{pseudo}}$	0.048	0.89	ϕ_{SV}	0.20	0.22
Osim	0.017	0.01	ϕ_{SL}	0.56	0.57
$\text{Otrue}_{\text{pseudo}}$	0.101	0.94	ϕ_{SG}	0.21	0.36
Vsim	0.078	1.7e-34	ϕ_{OV}	0.16	0.54
$\text{Vtrue}_{\text{pseudo}}$	0.132	0.86	ϕ_{OL}	1.91	0.58
Lsim	0.006	0.49	ϕ_{OG}	0.24	0.50
$\text{Ltrue}_{\text{pseudo}}$	-0.056	0.95	ϕ_{VL}	0.74	0.41
Gsim	0.034	6.6e-11	ϕ_{VG}	-0.35	0.02
$\text{Gtrue}_{\text{pseudo}}$	0.042	0.92	ϕ_{LG}	0.15	0.89

To assess the overall model fit
Model $\chi^2 = 1.59$ df $= 1$ p-value $= 0.21$
$GFI = 0.99$, $AGFI = 0.90$, $SRMR = 0.031$

For the remaining 9 data sets, the estimation algorithm failed to converge to a solution.

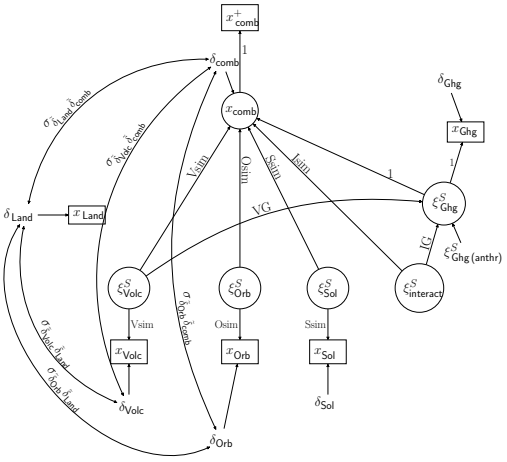

Figure S6.6.1. *Path diagram of the final SEM model fitted to the* Australasia *data in Step 1.*

Table S6.6.4. *The result of estimating the SEM model, depicted in Figure S6.6.1*

- *The result for Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.028	1.3e-13	$\sigma_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Land}}}$	2.6e-04	0.06
Osim	0.015	0.037	$\sigma_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{Volc}}}$	5.0e-04	0.08
Vsim	0.077	1.1e-34	$\sigma_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{comb}}}$	8.8e-04	5.6e-03
Isim	0.051	3.7e-05	$\sigma_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{comb}}}$	3.6e-04	0.13
VG	-0.0093	0.019	$\sigma_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{comb}}}$	9.4e-04	0.08
IG	-0.0103	0.1	$\text{Var}(\xi^S_{\text{Ghg (anthr)}})$	8.8e-04	3.9e-04

To assess the overall model fit

Model $\chi^2 = 5.9$ df $= 10$ p-value $= 0.82$

$GFI = 0.98$, $AGFI = 0.96$, $SRMR = 0.07$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}
x_{Sol}	-0.25	0.77	1.17	1.30	0.88	1.38
x_{Orb}		0.00	0.59	0.12	0.63	0.69
x_{Volc}			-0.04	-0.05	0.10	0.23
x_{Land}				0.00	0.76	0.31
x_{Ghg}					0.00	0.43
x^+_{comb}						0.52

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• *Summary of the results based on all 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\text{Ssim}}$	0.026	0.028	0.029	$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Land}}}$	2.6e-04	2.6e-04	2.6e-04
$\widehat{\text{Osim}}$	0.015	0.015	0.015	$\widehat{\sigma}_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{Volc}}}$	4.6e-04	4.9e-04	5.2e-04
$\widehat{\text{Vsim}}$	0.077	0.077	0.077	$\widehat{\sigma}_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{comb}}}$	7.5e-04	8.5e-04	9.5e-04
$\widehat{\text{Isim}}$	0.043	0.052	0.059	$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{comb}}}$	1.7e-04	3.4e-04	5.0e-04
$\widehat{\text{VG}}$	-9.5e-03	-9.2e-03	-8.9e-03	$\widehat{\sigma}_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{comb}}}$	5.5e-04	1.0e-04	1.4e-03
$\widehat{\text{IG}}$	-0.013	-0.011	-0.09	$\widehat{\text{Var}}(\xi^S_{\text{Ghg (anthr)}})$	8.1e-04	8.5e-04	9.1e-04
Model χ^2	5.6	6.0	6.8	GFI	0.98	0.98	0.98
p-value	0.66	0.74	0.78	AGFI	0.95	0.95	0.96
SRMR	0.064	0.067	0.071				

The solution for each data set is proper

Under the SEM model in Figure S6.6.1, $\text{Var}(\xi^S_{\text{Ghg}}) = \text{VG}^2 + \text{IG}^2 + \sigma^{2*}_{\text{Ghg}}$. For Data set number 10, $\widehat{\text{Var}}(\xi^S_{\text{Ghg}}) = 0.0011$ with the associated p-value 0.001. Hence, the standardised coefficient for ξ^S_{Ghg} is estimated as large as $\sqrt{\widehat{\text{Var}}(\xi^S_{\text{Ghg}})} = \widehat{\text{Gsim}} = 0.034$, which, based on p-value=0.001, is statistically significant at all conventional levels.

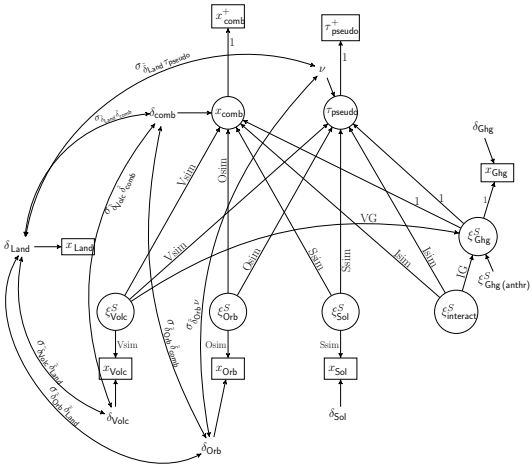

Figure S6.6.2. *Path diagram of the final SEM model fitted to the* Australasia *data in Step 2.*

Table S6.6.5. *The result of estimating the SEM model, depicted in Figure S6.6.2*

- *The result for Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.028	7.3e-14	$\sigma_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Land}}}$	2.7e-04	0.05
Osim	0.015	0.030	$\sigma_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{Volc}}}$	4.9e-04	0.10
Vsim	0.078	7.2e-36	$\sigma_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{comb}}}$	8.3e-04	6.9e-03
Isim	0.038	1.4e-05	$\sigma_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{comb}}}$	3.8e-04	0.10
σ^2_ν	0.0045	4.2e-10	$\sigma_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{comb}}}$	3.8e-04	0.35
$\text{Var}(\xi^S_{\text{Ghg (anthr)}})$	8.4e-04	1.7e-04	$\sigma_{\tilde{\delta}_{\text{Land}}\,\nu}$	6.8e-04	0.07
VG	-0.0094	0.043	$\sigma_{\tilde{\delta}_{\text{Orb}}\,\nu}$	9.1e-04	3.2e-03
IG	-0.013	0.036			

To assess the overall model fit

Model $\chi^2 = 7.6$ df $= 14$ p-value $= 0.91$

$GFI = 0.98$, $AGFI = 0.96$, $SRMR = 0.068$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}	τ^+_{pseudo}
x_{Sol}	-0.26	0.76	1.16	1.29	0.89	1.41	0.84
x_{Orb}		-0.05	0.58	0.08	0.63	0.62	0.37
x_{Volc}			-0.20	0.22	-0.43	0.40	0.56
x_{Land}				0.00	0.77	0.46	0.33
x_{Ghg}					0.14	-0.02	-0.05
x^+_{comb}						0.91	0.72
τ^+_{pseudo}							0.47

- *Summary of the results based on all 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\text{Ssim}}$	0.026	0.028	0.030	$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Land}}}$	2.6e-04	2.8e-04	3.5e-04
$\widehat{\text{Osim}}$	0.015	0.015	0.016	$\widehat{\sigma}_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{Volc}}}$	4.2e-04	5.9e-04	1.6e-03
$\widehat{\text{Vsim}}$	0.076	0.079	0.080	$\widehat{\sigma}_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{comb}}}$	7.2e-04	8.1e-04	1.0e-03
$\widehat{\text{Isim}}$	0.027	0.038	0.048	$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{comb}}}$	2.0e-04	3.5e-04	5.0e-04
$\widehat{\text{Var}}(\xi^S_{\text{Ghg (anthr)}})$	7.1e-04	8.0e-04	8.5e-04	$\widehat{\sigma}_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{comb}}}$	-1.7e-04	2.9e-04	7.9e-04
$\widehat{\text{IG}}$	-0.023	-0.017	-0.013	$\widehat{\sigma}_{\tilde{\delta}_{\text{Volc}}\,\nu}$	4.6e-04	8.3e-04	1.2e-03
$\widehat{\text{VG}}$	-0.012	-0.009	-0.006	σ^2_ν	0.0032	0.0042	0.0054
				$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\nu}$	-1.5e-04	4.4e-04	9.1e-03
Model χ^2	7.4	12.6	17.0	GFI	0.95	0.97	0.98
p-value	0.21	0.56	0.91	AGFI	0.90	0.93	0.96
SRMR	0.066	0.077	0.088				

The solution for each data set is proper

6.7 Antarctica (annual mean temperature)

Table S6.7.1. *The results of fitting the FA(2,1)-model and calculating the U_R- and U_T-test statistics. The results for the FA(2,1)-model and the U_R-statistic are based on the data sets $\{\bar{x}_{\text{single forcing}}, \bar{x}^{(i)}_{\text{comb}}\}$, where $\bar{x}^{(i)}_{\text{comb}}$ belongs to Data set number i, $i = 1, 2, \ldots, 10$, each of which is described in Table 1. The results for the U_T-statistic are based on the data sets $\{x_{\text{single forcing, repl.1}}, \ldots, x_{\text{single forcing, repl.k}}, x_u, \bar{x}^{(i)}_{\text{comb}}\}$, where x_u is the unforced climate model simulation*

- Single forcing: *Solar*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.067	<< 0.01	0.0162	<< 0.01	3.82	0.051	0.96	0.89	0.114	5.87	0.39

Summary of the results for all 10 data sets

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.064	0.014	1.82	0.019	0.95	0.85	0.080	5.29	0.224
Mean	0.066	0.016	3.30	0.083	0.97	0.90	0.105	5.71	0.462
Max	0.069	0.018	5.43	0.178	0.98	0.95	0.135	6.40	0.637

- Single forcing: *Orbital*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.079	<< 0.01	0.0144	<< 0.01	3.06	0.080	0.97	0.91	0.105	6.16	0.327

Summary of the results for all 10 data sets

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.075	0.013	0.99	0.08	0.97	0.91	0.059	5.26	0.31
Mean	0.078	0.015	2.08	0.16	0.98	0.94	0.086	5.80	0.50
Max	0.080	0.016	3.06	0.32	0.99	0.97	0.105	6.16	0.71

- Single forcing: *Volcanic*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.099	<< 0.01	0.010	<< 0.01	3.95	0.047	0.96	0.89	0.105	10.88	0.165

Summary of the results for all 10 data sets

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.096	0.009	1.25	0.019	0.95	0.84	0.061	10.0	-0.31
Mean	0.099	0.010	3.62	0.090	0.97	0.90	0.099	10.8	0.23
Max	0.101	0.011	5.55	0.265	0.99	0.96	0.124	11.2	0.58

- Single forcing: *Land*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.059	<< 0.01	0.0179	<< 0.01	2.73	0.10	0.97	0.92	0.104	3.96	0.82

Summary of the results for all 10 data sets

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.058	0.016	2.06	0.035	0.96	0.87	0.091	3.72	0.60
Mean	0.060	0.017	3.05	0.093	0.97	0.91	0.109	4.07	0.78
Max	0.062	0.019	4.47	0.151	0.98	0.94	0.132	4.47	1.03

- Single forcing: *Greenhouse gases (Ghg)*, $i = 10$

$\hat{\alpha}$	p-value	$\hat{\sigma}^2_\nu$	p-value	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
0.0415	< 0.01	0.021	< 0.01	0.275	0.60	0.99	0.99	0.03	0.99	1.87

Summary of the results for all 10 data sets

	$\hat{\alpha}$	$\hat{\sigma}^2_\nu$	Model χ^2	p-value	GFI	$AGFI$	$SRMR$	U_R	U_T
Min	0.040	0.019	0.066	0.45	0.994	0.983	0.016	0.65	1.50
Mean	0.041	0.021	0.316	0.59	0.997	0.991	0.034	0.98	1.89
Max	0.044	0.023	0.562	0.79	0.999	0.998	0.047	1.39	2.20

Table S6.7.2. *The final FA(6,5)-factor model obtained by simplifying the basic FA(6,6)-model from Table 2 (Antarctica)*

- *The result for Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.048	1.2e-08	Osim	0.061	3.8e-11
Vsim	0.090	8.4e-24	Lsim	0.031	0.02
Gsim	0.024	0.13	ϕ_{LG}	0.61	0.58

To assess the overall model fit
Model $\chi^2 = 61.23$ df $= 15$ p-value $= 1.6\text{e-}07$
$GFI = 0.80$, $AGFI = 0.71$, $SRMR = 0.212$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x_{comb}
x_{Sol}	1.00	3.88	4.22	3.97	1.41	3.06
x_{Orb}		0.62	2.62	2.79	1.65	2.56
x_{Volc}			0.14	2.51	-0.48	1.72
x_{Land}				1.11	1.86	2.71
x_{Ghg}					1.53	0.15
x_{comb}						0.60

- Summary of the results based on all 10 data sets

	Min	Mean	Max		Min	Mean	Max
$\widehat{\mathsf{Ssim}}$	0.044	0.047	0.050	$\widehat{\mathsf{Osim}}$	0.055	0.059	0.061
$\widehat{\mathsf{Vsim}}$	0.086	0.090	0.093	$\widehat{\mathsf{Lsim}}$	0.028	0.033	0.038
$\widehat{\mathsf{Gsim}}$	0.020	0.024	0.029	$\widehat{\phi}_{\mathsf{LG}}$	0.59	0.64	0.74
Model χ^2	60	62	64	GFI	0.79	0.79	0.80
p-value	6.1e-08	1.3e-07	2.4e-07	AGFI	0.70	0.71	0.72
SRMR	0.209	0.211	0.213				

The solution for each data set is proper

Table S6.7.3. *The result of estimating the TLS-FA(6,5)-model from Table 3 (Antarctica)*

- *The result for Data set number 8*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.058	4.3e-09	ϕ_{SO}	0.84	6.2e-05
$\mathsf{True}_{\mathsf{pseudo}}$	-0.008	0.94	ϕ_{SV}	0.75	1.7e-06
Osim	0.069	9.8e-09	ϕ_{SL}	0.64	0.19
$\mathsf{Otrue}_{\mathsf{pseudo}}$	0.068	0.47	ϕ_{SG}	0.37	0.23
Vsim	0.091	1.4e-19	ϕ_{OV}	0.49	6.1e-03
$\mathsf{Vtrue}_{\mathsf{pseudo}}$	0.152	0.16	ϕ_{OL}	**1.16**	0.25
Lsim	0.032	0.11	ϕ_{OG}	0.46	0.14
$\mathsf{Ltrue}_{\mathsf{pseudo}}$	-0.068	0.58	ϕ_{VL}	0.28	0.45
Gsim	0.046	2.49e-04	ϕ_{VG}	-0.09	0.70
$\mathsf{Gtrue}_{\mathsf{pseudo}}$	0.094	0.49	ϕ_{LG}	0.65	0.17

To assess the overall model fit
Model $\chi^2 = 0.093$ df $= 1$ p-value $= 0.76$
$GFI = 0.99$, $AGFI = 0.99$, $SRMR = 0.005$

The estimation procedure failed to converge to a solution for Data sets number 3, 5, 7, 9, and 10. For the remaining five data sets, the solutions were improper (inadmissible) with at least one correlation coefficient estimated to be larger than 1.

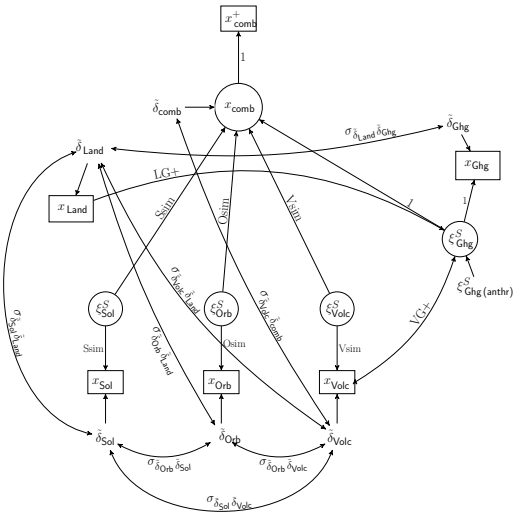

Figure S6.7.1. *Path diagram of the final SEM model fitted to the* Antarctica *data in Step 1.*

Table S6.7.4. *The result of estimating the SEM model depicted in Figure S6.7.1*

- *The result for Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.061	7.9e-12	$\sigma_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{Volc}}}$	0.0034	4.0e-04
Osim	0.072	4.6e-14	$\sigma_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{Land}}}$	0.0032	6.8e-05
Vsim	0.087	3.7e-25	$\sigma_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Sol}}}$	0.0033	1.1e-03
$\text{Var}(\xi^S_{\text{Ghg (anthr)}})$	0.0018	0.08	$\sigma_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Land}}}$	0.0030	1.5e-03
$\text{Var}(\tilde{\delta}_{\text{Land}})$	0.0092	9.1e-13	$\sigma_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Volc}}}$	0.023	0.035
LG+	0.571	7.8e-05	$\sigma_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{Land}}}$	0.0025	0.015
VG+	-0.163	0.06	$\sigma_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{comb}}}$	0.0035	8.6e-04
$\sigma_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{Ghg}}}$	-0.003	0.027			

To assess the overall model fit

Model $\chi^2 = 3.57$, df $= 6$, p-value$= 0.74$,
$GFI = 0.99$, $AGFI = 0.96$, $SRMR = 0.042$

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}
x_{Sol}	-0.31	0.03	0.57	-0.28	-0.36	0.71
x_{Orb}		-0.31	0.61	-0.42	0.07	0.63
x_{Volc}			0.41	0.04	0.13	0.35
x_{Land}				-0.11	-0.29	0.05
x_{Ghg}					0.48	-0.08
x^+_{comb}						0.21

to be continued on the next page

• *Summary of the results based on all 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\text{Ssim}}$	0.058	0.061	0.062	$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{Volc}}}$	3.2e-03	3.5e-03	3.7e-03
$\widehat{\text{Osim}}$	0.068	0.071	0.074	$\widehat{\sigma}_{\tilde{\delta}_{\text{Sol}}\,\tilde{\delta}_{\text{Land}}}$	3.0e-03	3.1e-03	3.2e-03
$\widehat{\text{Vsim}}$	0.086	0.087	0.089	$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Sol}}}$	3.2e-03	3.3e-03	3.5e-03
$\text{Var}(\xi^S_{\text{Ghg (anthr)}})$	0.0015	0.0018	0.0020	$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Land}}}$	2.8e-03	2.9e-03	3.0e-03
$\text{Var}(\tilde{\delta}_{\text{Land}})$	0.0091	0.0092	0.0092	$\widehat{\sigma}_{\tilde{\delta}_{\text{Orb}}\,\tilde{\delta}_{\text{Volc}}}$	2.3e-03	2.5e-03	2.8e-03
$\widehat{\text{LG+}}$	0.52	0.57	0.62	$\widehat{\sigma}_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{Land}}}$	2.4e-03	2.5e-03	2.6e-03
$\widehat{\text{VG+}}$	-0.172	-0.159	-0.147	$\widehat{\sigma}_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{comb}}}$	2.7e-03	3.4e-03	3.9e-03
$\widehat{\sigma}_{\tilde{\delta}_{\text{Land}}\,\tilde{\delta}_{\text{Ghg}}}$	-3.1e-03	-2.8e-03	-2.5e-03				
Model χ^2	1.0	2.9	5.4	GFI	0.98	0.99	1.00
p-value	0.49	0.80	0.99	AGFI	0.94	0.97	0.99
SRMR	0.022	0.037	0.049				

The solution for each data set is proper

Under the SEM model in Figure S6.7.1,

$$\widehat{\text{Var}}(\xi^S_{\text{Ghg}}) = \text{Var}(\xi^S_{\text{Ghg (anthr)}}) + (\text{LG+}) \cdot \Big((\text{VG+}) \cdot \sigma_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{Land}}} + (\text{LG+}) \cdot \text{Var}(\tilde{\delta}_{\text{Land}}) \Big) +$$
$$+ (\text{VG+}) \cdot \Big((\text{LG+}) \cdot \sigma_{\tilde{\delta}_{\text{Volc}}\,\tilde{\delta}_{\text{Land}}} + (\text{VG+}) \cdot \sigma^{2\,*}_{\text{Volc}} \Big) + (\text{VG+})^2 \cdot \text{Vsim}^2.$$

For Data set number 10, $\widehat{\text{Var}}(\xi^S_{\text{Ghg}}) = 0.0047$ with the associated p-value 0.041. Thus, the standardised coefficient for ξ^S_{Ghg} is estimated to be as large as $\sqrt{\widehat{\text{Var}}(\xi^S_{\text{Ghg}})} = \widehat{\text{Gsim}} = 0.069$, which, based on p-value=0.041, is statistically significant only at 5% level.

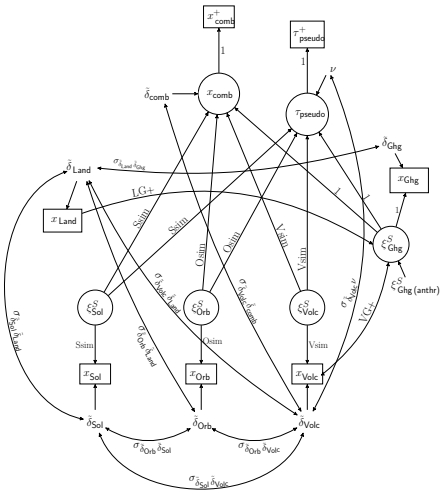

Figure S6.7.2. *Path diagram of the final SEM model fitted to the* Antarctica *data in Step 2.*

Table S6.7.5. *The result of estimating the final SEM model depicted in Figure S6.7.2*

• *The result for Data set number 10*

Parameter	Estimate	p-value	Parameter	Estimate	p-value	Parameter	Estimate	p-value
Ssim	0.061	1.4e-14	LG+	0.572	7.1e-05	$\sigma_{\tilde{\delta}_{\mathsf{Sol}}\,\tilde{\delta}_{\mathsf{Volc}}}$	0.0036	1.7e-04
Osim	0.074	1.1e-15	VG+	-0.148	0.08	$\sigma_{\tilde{\delta}_{\mathsf{Sol}}\,\tilde{\delta}_{\mathsf{Land}}}$	0.0032	7.3e-05
Vsim	0.088	7.1e-21	$\sigma_{\tilde{\delta}_{\mathsf{Land}}\,\tilde{\delta}_{\mathsf{Ghg}}}$	-0.003	0.025	$\sigma_{\tilde{\delta}_{\mathsf{Orb}}\,\tilde{\delta}_{\mathsf{Sol}}}$	0.0035	7.9e-04
$\mathrm{Var}(\xi^S_{\mathsf{Ghg}\,(\mathsf{anthr})})$	0.0018	0.085	σ^2_ν	0.0205	1.3e-09	$\sigma_{\tilde{\delta}_{\mathsf{Orb}}\,\tilde{\delta}_{\mathsf{Land}}}$	0.0031	1.2e-03
$\mathrm{Var}(\tilde{\delta}_{\mathsf{Land}})$	0.0092	9.0e-13	$\sigma_{\tilde{\delta}_{\mathsf{Volc}}\,\tilde{\delta}_{\mathsf{Land}}}$	0.0025	0.013	$\sigma_{\tilde{\delta}_{\mathsf{Orb}}\,\tilde{\delta}_{\mathsf{Volc}}}$	0.024	0.033
$\sigma_{\tilde{\delta}_{\mathsf{Volc}}\,\tilde{\delta}_{\mathsf{comb}}}$	0.0033	2.2e-03	$\sigma_{\tilde{\delta}_{\mathsf{Volc}}\,\nu}$	0.0041	4.3e-03			

To assess the overall model fit

Model $\chi^2 = 8.072$, df $= 11$, p-value $= 0.71$,
$GFI = 0.98$, $AGFI = 0.95$, $SRMR = 0.050$.

Normalised Residuals

	x_{Sol}	x_{Orb}	x_{Volc}	x_{Land}	x_{Ghg}	x^+_{comb}	τ^+_{pseudo}
x_{Sol}	-0.32	-0.07	0.36	-0.27	-0.38	0.68	-0.17
x_{Orb}		-0.45	0.55	-0.47	0.01	0.43	0.71
x_{Volc}			0.35	0.01	-0.06	0.30	0.71
x_{Land}				-0.12	-0.31	0.02	0.06
x_{Ghg}					0.56	-0.16	-0.64
x^+_{comb}						-0.01	0.87
τ^+_{pseudo}							0.85

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- *Summary of the results based on all 10 data sets*

	Min	Mean	Max		Min	Mean	Max
$\widehat{\text{Ssim}}$	0.058	0.061	0.062	$\widehat{\text{LG+}}$	0.53	0.56	0.59
$\widehat{\text{Osim}}$	0.067	0.070	0.074	$\widehat{\text{VG+}}$	-0.173	-0.154	-0.125
$\widehat{\text{Vsim}}$	0.085	0.087	0.089	$\widehat{\sigma}_{\bar{\delta}_{\text{Land}}\,\bar{\delta}_{\text{Ghg}}}$	-3.1e-03	-2.8e-03	-2.5e-03
$\text{Var}(\xi^{S}_{\text{Ghg (anthr)}})$	0.0016	0.0018	0.0020	$\widehat{\sigma}_{\bar{\delta}_{\text{Volc}}\,\bar{\delta}_{\text{comb}}}$	2.7e-03	3.4e-03	3.9e-03
$\text{Var}(\bar{\delta}_{\text{Land}})$	0.0091	0.0092	0.0092	$\widehat{\sigma}_{\bar{\delta}_{\text{Volc}}\,\nu}$	1.5e-03	3.4e-03	5.2e-03
$\widehat{\sigma}^2_{\nu}$	0.0161	0.0209	0.0275	$\widehat{\sigma}_{\bar{\delta}_{\text{Orb}}\,\bar{\delta}_{\text{Land}}}$	2.7e-03	2.9e-03	3.1e-03
$\widehat{\sigma}_{\bar{\delta}_{\text{Sol}}\,\bar{\delta}_{\text{Volc}}}$	3.2e-03	3.5e-03	3.7e-03	$\widehat{\sigma}_{\bar{\delta}_{\text{Sol}}\,\bar{\delta}_{\text{Land}}}$	3.0e-03	3.1e-03	3.3e-03
$\widehat{\sigma}_{\bar{\delta}_{\text{Orb}}\,\bar{\delta}_{\text{Sol}}}$	3.1e-03	3.3e-03	3.5e-03	$\widehat{\sigma}_{\bar{\delta}_{\text{Volc}}\,\bar{\delta}_{\text{Land}}}$	2.0e-03	2.4e-03	2.6e-03
$\widehat{\sigma}_{\bar{\delta}_{\text{Orb}}\,\bar{\delta}_{\text{Volc}}}$	2.2e-03	2.5e-03	2.8e-03				
Model χ^2	4.1	7.6	13.0	GFI	0.96	0.98	0.99
p-value	0.29	0.74	0.97	AGFI	0.91	0.95	0.97
SRMR	0.038	0.046	0.057				

The solution for each data set is proper

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