Title: On functions of a Wishart matrix and a normal vector with applications

Abstract

This thesis consists of two papers which take a critical look on functions of an inverse Wishart matrix and a Gaussian vector. In the first paper, the product expression, of which the inverse of the pooled estimator of the covariance matrix is inverse Wishart distributed and the difference of sample means is multivariate normally distributed, is investigated by exploring the distributional properties via a stochastic representation for both the finite sample case and the infinite sample case under the large-dimensional asymptotic regime. A test theory is developed to study the contribution of population coefficients in the discriminant function. Furthermore, we investigate the performance of the classification analysis based on the discriminant function.

In the second paper, the expression of the product of the inverse sample covariance matrix and the sample mean vector with regard to the derivation of moments of the estimated tangency portfolio weights is studied. In particular, higher order central and non-central moments of the elements of such a product are obtained. Moreover, the closed-form expressions of the second, the third and the fourth central moments are provided whose expressions do not depend on special mathematical functions, like the confluent hypergeometric function. Furthermore, we deduce analytical expressions for the mean, the variance, the skewness, and the kurtosis from the higher-order central and non-central moments of the tangency portfolio weights. Finally, the developed results have been successfully applied to real weekly data of eight stocks traded on NASDAQ.