Multiparty Quantum Communication and fs-laser Written Integrated Optics Circuits

Ashraf Mohamed El Hassan
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Abstract
Quantum information science, the rapidly developing interdisciplinary field, gives power to the information and communications technologies (ICT) by providing secure communication, precision measurements, ultra-powerful simulation and ultimately computation. It is well known that photons are an ideal candidate for encoding the quantum bit, or "qubit", in quantum information and specially for quantum communication. This thesis consists of two main parts. In the first part, realization of quantum security tasks using optical fibers has been implemented. Bell tests are a cornerstone of quantum key distribution and are necessary for device-independent security. Device-independent Bell inequality test must be performed with care to avoid loopholes. Time-energy entanglement has a distinct advantage over polarization as it is easier transmitted over longer distances, therefore, it may be preferable as a quantum resource to perform reliable key distribution. Novel multi-party communication protocols: secret sharing, detectable Byzantine agreement, clock synchronization, and reduction of communication complexity, all these quantum protocols has been realized without compromising on detection efficiency or generating extremely complex many-particle entangled states. These protocols are realized in an optical fiber setup with sequential phase modulation on single photons. In recent years there has been great interest in fabricating ICT optical setups in low scale in glass chips, which would replace the bulk setups on tables used today. In the second part of the thesis, realization of photonic waveguides in glass has been implemented. Using femtosecond laser inscription of waveguides in glass, photonic quantum technologies and integrated optical circuits are becoming more and more important in miniaturization of optical circuits written in different glass samples for the quantum optics and quantum information processing. The study and optimization the different building blocks for integrated photonic quantum circuits, for instance the directional coupler and Mach-Zehnder interferometer is very important. The principal goal is to develop a method for design, fabrication and characterization of integrated optics circuits for further applications in quantum information. Incorporation of photon sources, detectors, and circuits integrating waveguides technology can be used to produce integrated photonics devices.

Keywords: Quantum optics, quantum communication, quantum cryptography, fs-laser written waveguides, integrated optics circuits.

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This thesis consists of two main parts. In the first part, realization of quantum security tasks using optical fibers has been implemented. Bell tests are a cornerstone of quantum key distribution and are necessary for device-independent security. Device-independent Bell inequality test must be performed with care to avoid loopholes. Time-energy entanglement has a distinct advantage over polarization as it is easier transmitted over longer distances, therefore, it may be preferable as a quantum resource to perform reliable key distribution. Novel multi-party communication protocols: secret sharing, detectable Byzantine agreement, clock synchronization, and reduction of communication complexity, all these quantum protocols has been realized without compromising on detection efficiency or generating extremely complex many-particle entangled states. These protocols are realized in an optical fiber setup with sequential phase modulation on single photons. In recent years there has been great interest in fabricating ICT optical setups in low scale in glass chips, which would replace the bulk setups on tables used today. In the second part of the thesis, realization of photonic waveguides in glass has been implemented. Using femtosecond laser inscription of waveguides in glass, photonic quantum technologies and integrated optical circuits are becoming more and more important in miniaturization of optical circuits written in different glass samples for the quantum optics and quantum information processing. These platforms offer stability over the time-scales required for multi-photon coincidence based measurements. The study and optimization the different building blocks for integrated photonic quantum circuits, for instance the directional coupler and Mach-Zehnder interferometer is very important. The principal goal is to develop a method for design, fabrication and characterization of integrated optics circuits for further applications in quantum information. Incorporation of photon sources, detectors, and circuits integrating waveguides technology can be used to produce integrated photonics devices.
This thesis is dedicated to my family.
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PAPER I: "Hacking the Bell test using classical light in energy-time entanglement-base quantum key distribution"
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PAPER II: "Experimental quantum multiparty communication protocols"
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PAPER III: "Tunable integrated devices for quantum optics experiments using Fs laser writing Waveguides in glass"
Ashraf Mohamed El Hassan, Guillermo Andler, and Mohamed Bourennane.
Manuscript to be submitted (2019).

PAPER IV: "Corner states of light in photonic waveguides"**
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Author’s contribution

PAPER I: I made a major contribution in designing the experimental setup. I performed all the experimental work, which included the experimental setup and performing the measurements. I analysed the data together with Jonathan Jogenfors, and I contributed in writing the paper.

PAPER II: I contributed to the design of the experiment. I and Massimiliano Smania built the setup, measured, and analysed the data, and I contributed in writing the paper.

PAPER III: I optimized the fs-laser writing parameters. I wrote the circuits program. I designed and performed the experiment, I analysed the results and I wrote the paper with my colleagues.

PAPER IV: I optimized the fs-laser writing parameters. I wrote the circuits program. I designed, performed the experiment, analysed the data with the help of experimental co-authors and I contributed in writing the paper.
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1. Introduction

1.1 Overview

Quantum optics is an interdisciplinary field connecting quantum physics, optics, computer science, quantum information, and many other fields. The research implemented using light has advantages because photons travel with the speed of light and are weak coupled to the environment, so they are suited for quantum communication tasks. Quantum optics enables coherent control of a quantum optical system, makes quantum engineering possible, e.g., quantum interferometers go far beyond classical ones, that enable us to build applications, like tests of fundamental quantum mechanics in fiber-based systems and recently in optical integrated circuits.

Photonic quantum information using fibers make processes applicable over long distances, e.g., Quantum key distributions protocol tasks can be run in fibers over kilometres. It is well known that much work has been done in quantum information allowing us to test the completeness of quantum mechanics, i.e., violation of Bell inequality[1], to ensure that the system of communication is secure. Many tasks in communication, computation, and cryptography can be extended beyond classical limitations by using quantum resources. Quantum technologies often rely on strongly correlated particles that can be distributed and cannot be described using classical theory, i.e., violation of Bell inequality, in which the parties involved must share an entangled quantum state and perform suitable local measurements whose outcomes can be locally processed and communicated by classical means. In this thesis, an optical fiber setup has been studied showing one scenario for faking the test of Bell’s inequality using classical light instead, in such a way that an eavesdropper can have access to the devices and get the information without leaving a trace. I reported a single quantum system transition-assisted information processing tasks, i.e., secret sharing, detectable Byzantine agreement, and reduction of communication complexity using an optical fiber system.

Recently photonic quantum technologies in integrated optical circuits are becoming more and more important in the miniaturization of optical circuits written in different glass samples for quantum optics and quantum information processing. These platforms offer stability over the time-scales required...
for multi-photon coincidence-based measurements. The advantages of manufacturing waveguides in glass are that glass material has low cost, composition flexibility, easy doping, and low absorption in visible and infrared regimes. Moreover, waveguides written in glass have low propagation losses (0.1 – 0.5 dB/cm), excellent mode matching with fibers, low refractive index change, that makes excitation a single mode easy as well as the amorphous glass gives low intrinsic birefringence due to waveguides shape and stresses.

1.2 Thesis work

The studies included in this thesis were carried out in two types of photonic systems, i.e., a fiber-based system (first half of the PhD), and integrated circuits in glass (second half of the PhD).

In the first part using fibers: The project goal is an experimental implementation of photonic quantum information protocols and processing tasks, such as secure multi-party quantum communication, entanglement-assisted communication. We implemented novel multi-party quantum communication protocols: secret sharing, detectable Byzantine agreement, clock synchronization, and reduction of communication complexity. It will be shown that the quantum protocols outperform any classical one. The method used in the research was to use multipath interferometers using single mode fiber technology, single photon sources, and single photon detectors.

In the second part, a femtosecond laser writing system (a newly installed lab in our research group) was used to fabricate integrated photonic quantum circuits. That’s why the main goal of the studies in this part is to optimize the different building blocks for integrated Photonic quantum circuits. The system for producing optical circuits in glass is based on a femtosecond IR laser (300-350 fs; 3W), optical laser guiding formed by mirrors and Pinholes, for alignment; a focusing system determining the size and form of the inscribed waveguides, motion control stages with 1 nm precision, steered by a software, where we can draw the optical circuit to be studied. The substrate can be translated with respect to the focus, thereby forming lines of refractive index change which can act as waveguides. The laser can be focused at multiple depths inside the sample meaning that truly 3D waveguides circuits can be formed, with high transmission. The characterization of optical waveguides and circuits in glasses is done with 5-axis translation stages, where we measure losses and perform couplings. The glass substrate type used for optical circuits is alumino-borosilicate Corning EAGLE 2000, because of its low birefringence. Finally, fiber couplings for input and output signals for different optical circuits have to be implemented. The emerging strategy to overcome the limitations of bulk quantum optics consists of taking advantage of the ro-
bustness and compactness achievable by integrated waveguides technology.

1.3 Thesis outlines

I carried out this thesis in the quantum optics and quantum information group at the Department of Physics at Stockholm University. In the first chapter quantum optics, and quantum information will be introduced, in chapter two there is a brief introduction to quantum mechanics, illustrations to physical quantum states, measurement operators, the Poissonian statistics that gives a description of photonic quantum optics system measurements, also in this chapter Bell’s inequality, and quantum cryptography will be presented. Finally, sections give background on the realization of optical experiments and its building structures elements for integrated circuits will be presented. Then moving to the experiment part, chapter three introduces fiber optics based systems; experimental components that can be found in modern optics communication setups, used in performing quantum information tasks, and their working mechanism. In chapter four it will be shown how eavesdropper can control the Avalanche photodiode detectors, this shows one scenario of hacking quantum key distribution tasks. Chapter five presents how experiments for novel multiparty quantum communication protocols are carried out.

The second part of this thesis regards the integrated optics in glass using femtosecond laser writing techniques. In chapter six there will be an introduction of the laser writing technique, first I will introduce the fs-laser, then describe the interaction of the laser beam and the transparent material, after that I described how one can design a circuit and the lab types of equipment required, i.e., focusing system, 3D translation stage, and polishing machine. Also in this chapter, the methods used to characterize the inscribed waveguides will be introduced. This introductory chapter will be ended by an experimental work showing the fabrication steps and the optimization, analysis, and the basic integrated waveguide devices in borosilicate glass, i.e., straight waveguides, bending in waveguides. Finally, in last two chapters the applications and experiments done using the integrated circuits, i.e., directional coupler, Mach-Zehnder interferometers, and the study of a photonic topological insulator system will be presented.

The last part of the thesis contained scientific papers and the manuscripts which show the research outcome for this Doctoral degree in physics. The research projects of this thesis were supported by grants from Swedish research council (VR) and Knut and Alice Wallenberg foundation (KAW).
2. Concepts and background

Quantum optics is a relatively new field in practice, although the theory is old as the quantum theory itself. In quantum optics optical phenomenon is treated by a stream of photons rather than as electromagnetic waves. Recently it has been involved in many practical implementations of technologies, e.g., quantum information and test of the foundation of quantum theory [2].

2.1 Concepts from Quantum information

2.1.1 The qubit

A physical system in quantum mechanics is completely described by a state vector $|\psi\rangle$, in a Hilbert space $\mathcal{H}$ (A space that contains arbitrary but finite dimensions). A pure state is the state that can describe the system, using a single state vector. The fundamental element of a quantum system is the quantum bit (qubit), which corresponding to classical bit in classical information theory. A qubit is a state in a two dimensional Hilbert space. It can be a superposition of any two basis states (in Dirac notation):

$$|Q\rangle = c_1|0\rangle + c_2|1\rangle,$$

where $c_1$ and $c_2$ are complex numbers, the probabilities satisfies the normalization condition.

$$|c_1|^2 + |c_2|^2 = 1.$$  \hfill (2.2)

The qubit state can be represented and graphically visualized by a point on the surface of Bloch sphere as:

$$|Q\rangle = \cos \left( \frac{\theta}{2} \right) |0\rangle + e^{i\phi} \sin \left( \frac{\theta}{2} \right) |1\rangle,$$

where $0 \leq \phi \leq 2\pi$ and $0 \leq \theta \leq \pi$. The orthogonal states are opposite in the Bloch sphere so that:

$$|Q(\theta, \phi)\rangle_{orthogonal} = |Q(\pi - \theta, \pi + \phi)\rangle.$$ \hfill (2.4)
The quantum bit state can also be represented using vectors:

\[ |0\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |1\rangle \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{2.5} \]

then the qubit can be expressed as a superposition of these vectors:

\[ |Q\rangle = c_0 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}. \tag{2.6} \]

Any operator \( \hat{O} \) act on a single qubit can be represented in a matrix form as a combination of Pauli matrices and the identity matrix:

\[
\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{2.7}
\]

and for any Hermitian \( 2 \times 2 \) matrix, the operator \( \hat{O} \) can be written as:

\[ \hat{O} = c_x \sigma_x + c_y \sigma_y + c_z \sigma_z + c_1 I, \tag{2.8} \]

where the coefficients \( c_1, c_2, c_3 \) and \( c_1 \) are real values.

2.1.2 Mixed states and the density operators

A mixed state can be described using a density operator. The density operator is a sum of pure states \( |\psi_i\rangle \) with probabilities \( p_i \) of the system, and can be written as follow:

\[ \rho = \sum_i p_i |\psi_i\rangle \langle \psi_i| . \tag{2.9} \]

The density operator has two properties which are important for quantum measurements and the definition of operation.

\[ Tr(\rho) = 1, \tag{2.10} \]

Figure 2.1: Bloch sphere.
2.1. Concepts from Quantum Information

\[ \rho^\dagger = \rho. \] (2.11)

These conditions mean: any density matrix has to be normalized (its traces are equal to one), and the density matrix \( \rho \) is a positive operator. The positivity of the density matrix implies that it is Hermitian. For the pure state the density matrix can be reduced to:

\[ \rho = |\psi\rangle \langle \psi|. \] (2.12)

Pure state, contain maximal knowledge of the system and it has the following properties:

\[ \rho^2 = \rho, \] (2.13)

\[ Tr(\rho^2) = 1, \] (2.14)

where the trace is the summation along the diagonal matrix elements. This trace value is less than 1 in the case of mixed states. The expectation value of an operator \( \hat{O} \) acting on a quantum state \( \rho \) (pure or mixed) is the trace product over the operator and the state density:

\[ E(\hat{O}) = Tr(\hat{O}\rho). \] (2.15)

2.1.3 Multi qubit systems

Multi qubit system, consisting of more than one qubit, is described by vectors in product Hilbert space for each qubit. For a system of two qubits, of a qubit space \( \mathcal{H}_1 \) and a qubit space \( \mathcal{H}_2 \) given by tensor product:

\[ \mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2. \] (2.16)

If the basis of each space can be written as:

\[ |0\rangle_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |1\rangle_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \] (2.17)

where \( i = 1, 2 \) for qubit 1 and qubit 2 respectively. The tensor product of individual qubit spaces can expressed as follow:

\[ |00\rangle \equiv |0\rangle_1 \otimes |0\rangle_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad |01\rangle \equiv |0\rangle_1 \otimes |1\rangle_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \] (2.18)
A product of two local operators in this context can be described by the Kronecker product in the following way:

\[
\hat{A} \otimes \hat{B} = \begin{bmatrix} a_{11} \hat{B} & a_{12} \hat{B} \\ a_{21} \hat{B} & a_{22} \hat{B} \end{bmatrix}.
\] (2.20)

This can be generalized to higher dimensions. The dimension of an N composite qubits system will be \(2^N\).

### 2.1.4 Mutually unbiased bases (MUBs) and distinguishability

MUBs can be defined as a set of vectors or a set of matrices. Any two orthonormal bases \(|e_i⟩, ..., |e_d⟩\) and \(|f_i⟩, ..., |f_d⟩\) are said to be mutually unbiased bases if the square of the magnitude of the inner product between any vector from the first basis \(|e_i⟩\) and any vector from the second basis \(|f_j⟩\) obey:

\[
|⟨e_i|f_j⟩|^2 = \frac{1}{d},
\] (2.21)

for \(d\) dimension Hilbert space. That means if a state has been prepared on one basis and then measured using a mutually unbiased basis, likely all outcome will be detected with equal probabilities. The first basis knowledge implies ignorance of the second (e.g., Heisenberg’s uncertainty relation for position and momentum in infinite dimensional case). Since the outcome is random mutually unbiased bases are useful in experimental scenarios when applied in many protocols, e.g., in quantum key distribution. For any normalized eigenvectors \(|\psi^a_i⟩\) of \(\hat{A}\) and \(|\psi^b_j⟩\) of \(\hat{B}\) using 2.21:

\[
|⟨\psi^a_i|\psi^b_j⟩| = \frac{1}{\sqrt{d}}.
\] (2.22)

There will be a set of \(M = d + 1\) mutually complementary bases if \(d = p^k\), where \(p\) is a prime and \(k\) an integer [3]. One can choose arbitrary the first basis as

\[
|\psi^a_0⟩, |\psi^a_1⟩, ..., |\psi^a_{d-1}⟩,
\] (2.23)

where \(|⟨\psi^a_k|\psi^a_l⟩| = δ_{kl}\). For two complementary bases, the second basis will be chosen as

\[
|\psi^b_k⟩ = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} exp\left(\frac{2\pi i kn}{d}\right)|\psi^a_n⟩.
\] (2.24)

\[
|10⟩ ≡ |1⟩_1 \otimes |0⟩_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad |11⟩ ≡ |1⟩_1 \otimes |1⟩_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.
\] (2.19)
2.1. Concepts from Quantum information

It is mathematically hard to find the MUBs for any dimension. For $d=2$ they are correspond to horizontal/vertical, diagonal/anti-diagonal and left/right circular polarization states, given below in computational basis $B_1, B_2$ and $B_3$ respectively:

$$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad B_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}. \quad (2.25)$$

For $d=3$ three:

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_2 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}, \quad M_3 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & \omega & \omega \\ \omega & 1 & \omega \\ \omega & \omega & 1 \end{bmatrix}, \quad M_4 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & \omega^2 & \omega^2 \\ \omega^2 & 1 & \omega^2 \\ \omega^2 & \omega^2 & 1 \end{bmatrix}. \quad (2.26)$$

where $\omega = e^{\frac{2\pi i}{3}}$.

2.1.5 No-cloning theory

If a quantum state is prepared in one base and measured on the same basis the result of the measurement is distinct, otherwise, if the state is measured on other bases then the result is random. The no-cloning theorem stated that linearity of quantum mechanics did not allow copying ideally an unknown arbitrary pure quantum system [4]. One way to prove the no-cloning is to define a unitary operator $\hat{U}$ which can perform the operation below:

$$\hat{U}(\ket{\psi}_a \otimes \ket{B}_b) = \ket{\psi}_a \otimes \ket{\psi}_b. \quad (2.27)$$

The operation copies the qubit $\psi$ into other initially qubit in a blank state $\ket{B}$. Then it must be able to copy other states, e.g., $\ket{\phi}$ :

$$\hat{U}(\ket{\phi}_a \otimes \ket{B}_b) = \ket{\phi}_a \otimes \ket{\phi}_b. \quad (2.28)$$

Taking the inner product of equations above:

$$\langle B_b | \otimes \langle \psi_a | \psi \rangle \hat{U}^\dagger \hat{U}(\ket{\phi}_a \otimes \ket{B}_b). \quad (2.29)$$

The inner product will gives two expressions:

$$\langle \psi |_b \otimes \langle \psi |_a \phi \rangle \otimes \ket{\phi}_b = \langle \psi, \phi \rangle^2, \quad (2.30)$$

$$\langle B |_b \otimes \langle \psi |_a \phi \rangle \otimes \ket{B}_b = \langle \psi, \phi \rangle. \quad (2.31)$$
The two equations above are both fulfilled only if $\psi$ and $\phi$ are either equal or orthogonal. The perfect copying quantum state machine cannot exist, but of course there are imperfect copies machines.

For two dimensional systems the eavesdropper machine can copy photons in one of the basis, but the process will be detected by the key distributors due to the random signal disturbance caused by the inability to copy photons in the other basis. The fidelity (the "closeness" of two quantum states, it expresses the probability of one state that can be identified as the other) of the optimal cloning of two dimension systems in case of two copies, is equal to $\left(\frac{5}{7}\right)$ [5].

### 2.1.6 Bell inequality

In quantum mechanics, a system of two non-commuting physical properties cannot have a definite value. Probabilities of different measurements outcomes can only be provided by quantum mechanics. This was such a big concern for Einstein, Podolsky and Rosen, and the argument of quantum mechanics is not complete theory come out. Then later a hidden variable theory has been proposed, state that any particle has information about the result of measurement that can be made on it. Bell's theorem stated that the correlations by an entangled state that can be predicted using quantum mechanics, cannot properly be described by local hidden variable theories [1]. There are two assumptions in Bell's model:

**Realism:** All physical properties have definite values independent of observation. Therefore the physical objects properties pre-exist without the influence of the observable.

**Locality:** The measurements of one particle cannot have any influence on other measurements results in other space outside its light cone. That means no information can be transferred faster than the speed of light.

The version of Bell's inequality proposed by Clauser, Horne, Shimony, and Holt namely CHSH inequality, it can be derived as follow. Consider pairs of photons simultaneously emitted in opposite directions. They arrive at two very distant measuring devices A and B, at Alice and Bob stations, respectively. Their apparatuses have only two outputs, i.e., yes or no, as detection events on $+1$ or $-1$ Alice and Bob detectors.

Alice measures the properties $P_{a_1}$ and $P_{a_2}$, while Bob measures in his station the properties $P_{b_1}$ and $P_{b_2}$. The properties measurements outcomes for Alice are $A_1$ and $A_2$ having outcomes results values with either $+1$ or $-1$, and for Bob, the outputs of measuring the properties $B_1$ and $B_2$ are also either $+1$ or $-1$. By calculating the quantity:

$$A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 = (A_1 + A_2)B_1 + (A_1 - A_2)B_2 = \pm 2,$$

(2.32)
since:

\((A_1 + A_2) = \pm 2\) if \((A_1 - A_2) = 0\), and \((A_1 + A_2) = 0\) if \((A_1 - A_2) = \pm 2\), suppose the probabilities of the outcomes, \(A_1\), \(A_2\), \(B_1\), and \(B_2\) are \(a_1\), \(a_2\), \(b_1\), and \(b_2\) respectively. Then the expectation value of the left hand side of equation:

\[
|\langle A_1 B_1 + A_1 B_2 + A_2 B_1 - A_2 B_2 \rangle| = |(A_1 + A_2) B_1 + (A_1 - A_2) B_2)|
\]

\[
= | \sum_{a_1,a_2,b_1,b_2} p(a_1,a_2,b_1,b_2) (a_1 b_1 + a_1 b_2 + a_2 b_1 - a_2 b_2) |
\]

\[
= | \sum_{a_1,a_2,b_1,b_2} p(a_1,a_2,b_1,b_2) a_1 b_1 + \sum_{a_1,a_2,b_1,b_2} p(a_1,a_2,b_1,b_2) a_1 b_2
\]

\[
+ \sum_{a_1,a_2,b_1,b_2} p(a_1,a_2,b_1,b_2) a_2 b_1 - \sum_{a_1,a_2,b_1,b_2} p(a_1,a_2,b_1,b_2) a_2 b_2 |
\]

\[
= |\langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle| \leq 2.
\]

Suppose Alice and Bob share the maximally entangled Bell state:

\[
|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).
\]

having the observables and the measurements results as follow:

\[
A_1 = \sigma_x, \quad A_2 = \sigma_y, \quad B_1 = -\frac{\sigma_x - \sigma_y}{\sqrt{2}}, \quad B_2 = \frac{\sigma_y - \sigma_x}{\sqrt{2}}.
\]

Calculating the first expectation value in equation 2.33, i.e., far left:

\[
|\langle A_1 B_1 \rangle| = \langle \psi^- | A_1 B_1 | \psi^- \rangle
\]

\[
= -\langle \psi^- | \frac{\sigma_x \sigma_x}{\sqrt{2}} | \psi^- \rangle - \langle \psi^- | \frac{\sigma_x \sigma_y}{\sqrt{2}} | \psi^- \rangle
\]

\[
= -\frac{1}{\sqrt{2}}.
\]

similarly the calculations of the other terms in the equation 2.33 give:

\[
|\langle A_1 B_2 \rangle| = \frac{1}{\sqrt{2}}, \quad |\langle A_2 B_1 \rangle| = \frac{1}{\sqrt{2}}, \quad |\langle A_2 B_2 \rangle| = -\frac{1}{\sqrt{2}}.
\]

Using the long above equation the entangled state measurements results:

\[
|\langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle| = 2\sqrt{2}.
\]

The equation above tells us that in quantum mechanics, the Bell’s inequality can be violated which is impossible using classical physics.
Chapter 2. Concepts and background

2.2 Optical photonic experiments background

Full realization of quantum photonics building blocks and devices has three main build structures, i.e., single photons and entangled sources, quantum devices, and measurements components involving single photon detectors.

2.2.1 Single Photon sources

Single photons sources, in principle should emit exactly one photon in response to a trigger pulse. Such sources are highly needed to guarantee the security in quantum cryptography experiments, otherwise the light pulse will containing more than one photon, the eavesdropper then can use beam splitting attack and she get all the information without being detected.

Attenuated laser source

The single photons source that is used in our fiber communication experiments is a highly attenuated coherent light from a laser source. Standard semiconductors laser generate coherent states can be used as attenuated single photon source, such light source give an expectation of high value for single photons events while the possibility of having multi photons events will be relatively low because the photon number distribution in coherent light is Poissonian.

2.2.2 Light state transformation

To run an optical experiment we used a setup either using free space bulk optics, fibers, or integrated circuits in the chip. The realization with these systems using photonic qubits has a great advantage because it is relatively easy to generate, control, manipulate, and detect quanta of light.

Polarizers

The polarizers is an optical component that pass specific polarization (in the ideal case). Absorptive linear Polarizers type, which pass only the target polarization and absorbs the rest used in the experiments, it is preferable to have a high extinction ratio and a wide range of working wavelength. This device uses to set the photon in certain polarization (a degree of freedom where the qubits are to be encoded) and to change the mix states to pure states. For a transverse light wave (the electric field always orthogonal to the direction
of the propagation) the polarization of the transverse wave can be described by Jones vector [6]. Table 2.1 shows Jones matrices for some of the common polarizing states.

<table>
<thead>
<tr>
<th>Polarization</th>
<th>Jones matrices</th>
<th>Polarization</th>
<th>Jones matrices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>$\begin{bmatrix} 1 \ 0 \end{bmatrix}$</td>
<td>Vertical</td>
<td>$\begin{bmatrix} 0 \ 1 \end{bmatrix}$</td>
</tr>
<tr>
<td>Right Circular</td>
<td>$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \ i \end{bmatrix}$</td>
<td>Left Circular</td>
<td>$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \ -i \end{bmatrix}$</td>
</tr>
<tr>
<td>Diagonal (+45)</td>
<td>$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \ 1 \end{bmatrix}$</td>
<td>Anti-diagonal (-45)</td>
<td>$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \ -1 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

Table 2.1: Jones matrices for some common polarization degenerate states.

Linear polarizers used to prepare photons in pure state in optics labs. If a plane polarized light is incident on the analyser, the intensity of a light transmitted by the analyser (I) is directly proportional to the square of the cosine of an angle between the polarizer and the transmission axis of the analyser according to Malus Law as:

$$I(\theta) = I_o \cos^2(\theta),$$

(2.39)

$I_o$ is the intensity between the polarizer and analyser. Absorptive type polarizers can be made of elongated metallic (e.g., silver) nano-particles embedded in thin glass plates.

Wave-plates

The wave-plates are components that can manipulate the photon, it changes the light polarization that passes through it or produces a phase shift to them by choosing their orientations relative to the transmission light. Wave-plate are crystals made from birefringence material usually quartz that cut in a plane which the index of refraction of the light pass through the ordinary axis is different to the extraordinary one.

The two most used ones in our work are half-wave plates (HWPs) and quarter-wave plates (QWPs), which introduce phase shift of $\pi$ and $\frac{\pi}{2}$ respectively. The effect act can be described with the use of the coordinate system rotation matrix $R(\theta)$, and the phase shift matrix $P(\phi)$ by Jones calculus:

$$R(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix},$$

(2.40)
Section 2. Concepts and Background

$$P(\phi) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix},$$ (2.41)

where $\theta$ is the angle of rotation around the optical axis (the angle between the fast axis of the wave plate and the lab frame H-axis), and $\phi$ is the retardation of the slow axis compared to the fast axis of the wave plate. The matrix of the general wave plate $W(\theta, \phi)$ can be obtained by: First rotating the coordinate system so that the x-axis aligned to fast axis of the wave plate, second by adding the phase shift, and finally by rotating back the coordinate system. The matrix of the general wave plate can be described using Jones calculus as

$$W(\theta, \phi) = \begin{bmatrix} \cos^2(\theta) + e^{i\phi} \sin^2(\theta) & \frac{1}{2} \left(1 - e^{i\phi}\right) \sin(2\theta) \\ \frac{1}{2} (1 - e^{i\phi}) \sin(2\theta) & \sin^2(\theta) + e^{i\phi} \cos^2(\theta) \end{bmatrix}. \quad (2.42)$$

Beam splitters

The Beam splitter (BS) is a device that splits the input light to two or more outputs fields. A beam splitter (i.e., 50:50) can be used for experimentally verify the indistinguishable of two photons. Using quantum mechanics one can predicts that the two photons will always emerge on the same/different output (outputs), the reason because of the possibility for each photon to emerge on a certain output when using the beam splitter is given by a probability amplitude. The most common bulk BS uses in setups is a beam splitter cube which composed of two right angle triangular glass prisms glued together at their hypotenuse surface. The beam splitter outputs fields can be designed depended on its polarization, this device name a polarization beam splitter (PBS), in fiber based beam splitter, the device designed to split a single input into its orthogonal linear polarization through two fiber outputs.

2.2.3 Detectors

Quantum optics considers that the light is consists of a stream of photons rather than a wave in classical picture. Light beam detection usually uses photon counters, very sensitive light detectors, e.g., avalanche photodetectors. Such detectors produce a short voltage pulse corresponding to each photon of light that hit the detector, and the counter registers the number of counts within a certain time interval that can be set by the users.
2.2. Optical photonic experiments background

**Photons statistics**

At the detector, the average count rate is determined by the light beam intensity, while the actual count rate is suffering from fluctuations during the measuring process. The fluctuations in the counting rate give information about the statistical properties of the stream light, although it is not a simple argument, whether what is registered by the counter is related to the photon statistic or it is an artifact from the detection process. Poisson statistics applied to random variables have only a positive integer’s values. However, the precise result of the individual measurement is not predictable [2]. The Poisson distribution probability $p(N)$, for $N$ events is given by

$$p(N) = \frac{\mu^N}{N!} e^{-\mu},$$  \hspace{1cm} (2.43)

where $\mu$ is a constant representing the mean value of the detected photons per unit of time. The distribution means value of $n$ events can be given by:

$$\bar{N} \equiv \langle N \rangle = \sum_{N=0}^{\infty} N p(N) = \mu. \hspace{1cm} (2.44)$$

The standard deviation of the Poisson distribution is equal to the square root of the mean value:

$$\sigma = \sqrt{\bar{N}}. \hspace{1cm} (2.45)$$

The photon distribution for coherent light is Poissonian.

**Single photon detectors**

Single photon detectors used in the quantum communication application and experiments are avalanche photo-diodes detectors (APDs). APDs are semiconductor diode devices type based on a P-N junction, operated in Geiger mode, with reversed bias voltage higher than the breakdown voltage (VB), i.e., when a photon hit an APD, the absorbed photon will create an electron-hole pairs, the two charge carriers will then accelerated in opposite directions due to the high bias voltage (about several hundred volts). Both the free electron and the hole can break to more electron making more hole pairs, and thus lead to an avalanche current, which is easily detectable. The high current flow need to be ending, therefore the process triggers an active quenching circuit which works to decrease the bias below breakdown voltage to stop the avalanche. For the same reason (avoiding the high current in the internal circuits) APDs also work in brief time window opening only when expecting a photon. Later the device kept silent for a while, normally in ns (dead time) before it becomes ready again for the next event. The APDs used in our lab for
the Telecom wavelength 1550 nm are PGA-600 detectors (InGaAs avalanche photodiodes are used in its gated mode), from Princeton Light-wave. The single photon unit detector detects single Telecom-wavelength photons using an InGaAs/InP avalanche photo-diode (APD) biased above its reverse breakdown voltage “Geiger mode”. These APDs are subject to a combination of a sub-breakdown DC bias voltage and a short pulse (1 ns, 4 V amplitude) that shortly brings the overall APD bias above breakdown. A photon impinging on the APD during this brief time window can initiate an avalanche. This burst of electrons is detected by the internal electronics, which produces an output logic pulse corresponding to each detection event. The photon counting distribution for a detector with a quantum efficiency \( \eta < 1 \) is also Poissonian.
Part I

OPTICAL FIBRE QUANTUM
MULTI-PARTITE
COMMUNICATION AND
QUANTUM
CRYPTOGRAPHY
3. Optical fibers system

The first part of this work is dedicated to the implementation of quantum communication and quantum cryptography tasks using optical fiber systems.

3.1 Optical fiber

In telecom transmission, photons propagate through optical fibers. An optical fiber usually consists of a core of silica and a cladding. The core region in fiber technology has a higher refractive index than the cladding, which surrounds the core. Due to the different core of optical fiber in the refractive index, light propagates inside the optical fiber by the effect of total internal reflection.

3.1.1 Single mode fibers

There are two distinguishable types of fibers working in different modes, depending on the ratio between the diameter of the fiber core and propagation light wavelength, which is smaller in case of single mode fibers (SMF) and bigger for the multi-mode fibers (MMF).

![Cross-section of single and multi-mode fibers.](image)

**Figure 3.1:** Cross-section of single and multi-mode fibers.

In experiments, fibers can be under stress and bending can change the polarization of the propagating light, therefore one has to compensate this effect using a polarization controller or instead polarization maintaining fibers.
Chapter 3. Optical fibers system

(PMFs) can be used. Connections between fibers are made by mechanical cleaving making which makes the ends both flat and parallel to each other.

3.1.2 Refractive index

The refractive index, is defined by the equation:

\[ n = \frac{c}{v} = \sqrt{\varepsilon_r \mu_r}, \tag{3.1} \]

where \( v \) is the velocity of light in a medium with refractive index \( n \), \( \varepsilon_r \) and \( \mu_r \) are the relative dielectric permeability and Magnetic permittivity of the medium respectively.

3.1.3 Numerical aperture of the fiber

The numerical aperture (NA) is an important parameter in fibers and it can be calculated as [6]:

\[ NA = \sin \alpha = \sqrt{n_o^2 - n_c^2}, \tag{3.2} \]

where \( n_o \) and \( n_c \) are the refractive indices of core and the cladding regions.

\[ \text{Figure 3.2: Numerical aperture and total internal reflection in fibers.} \]

For optical fibers NA usually takes value from 0.1 to 0.4. It is appropriate to introduce the normalized wave number \( V \) see equation 3.3 below, which gives the relation for the radius of the core \( r \) in term of wavelength,

\[ V = \frac{2\pi r}{\lambda} NA. \tag{3.3} \]

Only the fundamental mode can propagate inside the fiber, if \( V \leq 2.405 \).

3.1.4 Group velocity

The velocity of the envelope of a light pulse propagating in a medium is called the group velocity and can be defined by the equation:

\[ v_g = \frac{\partial \omega}{\partial k}, \tag{3.4} \]
where $\omega$ the angular frequency and $k$ is the wave number of the light. While the light propagates inside optical fibers it faces a light dispersion, where considerable of pulse broadening occurs, because the different frequency components of the pulse propagate along with the medium having different delays (different colours undergo different delays). In other words, the chromatic dispersion results on different spectral components of the pulse to travel at slightly different group velocities making a group velocity dispersion. The equation below gives the group velocity dispersion (GVD) or simply fiber dispersion as:

$$GVD = \frac{d^2k}{d\omega^2} = \frac{\partial}{\partial \omega} \frac{1}{v_g}.$$  \hspace{1cm} (3.5)

### 3.1.5 Attenuation in fibers

The intensity of electromagnetic waves propagate inside fibers decreases exponentially with distance, due to absorption and scattering. This attenuation is normally expressed in Decibel per km, i.e., as:

$$\alpha(dB/km) = \frac{10}{L} \log_{10} \frac{P_{in}}{P_{out}},$$  \hspace{1cm} (3.6)

where $P_{in}$ and $P_{out}$ are the input power of the light, and the output power values receptively, while $L$ is the length of the fiber in km. Recent improvements in fiber purity have reduced attenuation losses on order of 0.1 dB/km [7].

### 3.2 Optical fiber system components

#### 3.2.1 Fiber couplers

The 2X2 fiber based coupler is the basic optical component, with two input ports and two output ports, the device ports support a single mode of a certain wavelength. The electric field strength at one of the four ports can be described

![Figure 3.3: 2x2 Optical coupler.](image)
as:

$$E_j(t) = \frac{1}{2} E_j e^{-iwt} + c.c,$$  \hspace{1cm} (3.7)

where $j$ is 1, 2, 3, and 4, and $c.c$ is the complex conjugate. The above equation gives the complex amplitudes of the electric fields, and can be written supposing a low loss coupler, using the unitary matrix $(U)$ as:

$$\begin{bmatrix} E_3 \\ E_4 \end{bmatrix} = U \begin{bmatrix} E_1 \\ E_2 \end{bmatrix},$$  \hspace{1cm} (3.8)

where the unitary matrix is equal:

$$U = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix},$$  \hspace{1cm} (3.9)

where $a$, $b$ are the complex values, and $a^*$, $b^*$ are their complex conjugates. The normalization constraints, that the coupler power splitting ratios for the parallel and cross-paths and the conserved power can be described with the two equations below:

$$|a|^2 + |b|^2 = 1,$$  \hspace{1cm} (3.10)

$$|E_3|^2 + |E_4|^2 = |E_1|^2 + |E_2|^2.$$  \hspace{1cm} (3.11)

Moreover, this unitary matrix could have phase factors, that change the transformation matrix, but a global phase factor has no significant change in the experiment analysis.

### 3.2.2 Optical phase modulators

The phase modulator is a device based on the electro optical effect that allows the phase of the light beam to be controlled electrically. The linear electro-optic effect is the change in the refractive index that is proportional to the magnitude of an externally applied electric field. The phase modulators based on Lithium niobate (LN) material, commercially can reach up to 40 GHz modulation frequency.

### 3.2.3 Optical fiber circulators

An optical circulator (Cir) is a device based on the electro-optical effect, which allows the signal paths in an optical circuit to be controlled. If light travelling in paths in opposite directions are needed, then port 3 is active as shown in fig 3.4.
3.3 Fiber based interferometers

The qubits are encoded using the relative phase between two light pulses; which are natural solution [8]. State preparation and analysis to make the communication protocols perform by interferometers using single mode optical fiber components.

3.3.1 Mach-Zehnder interferometer

The layout in Fig.3.5 shows a simple version of a Mach-Zehnder (MZ) interferometer (Balanced version) that contains two 50:50 couplers ($C_1$ and $C_2$) and a phase shift modulation element in between. The Mach-Zehnder interferometer, can represented using matrices. $M_1$ and $M_3$ describe the couplers:

$$M_1 = M_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \quad (3.12)$$

$M_2$ describe the phase shift

$$M_2 = \begin{bmatrix} e^{i\frac{\phi}{2}} & 0 \\ 0 & e^{-i\frac{\phi}{2}} \end{bmatrix}. \quad (3.13)$$
This led to an overall transformation:

\[
M_T = M_3 M_2 M_1 = \begin{bmatrix}
\cos\left(\frac{\phi}{2}\right) & \sin\left(\frac{\phi}{2}\right) \\
\sin\left(\frac{\phi}{2}\right) & \cos\left(\frac{\phi}{2}\right)
\end{bmatrix}.
\] (3.14)

Which in the case of \( \phi = 0 \) gives 100% parallel-state, and a 100% cross-state for \( \phi = \pi \). In addition, the unbalanced Mach-Zehnder interferometer contains a loop delay \( \Delta T \) which if a train of pulses propagate inside the interferometer, by modulation one can produce output interference independence of the choices of the phase.

Beside the Mach-Zehnder interferometer that contains two 50/50 couplers, a three-path Mach-Zehnder interferometer with \( 3 \times 3 \) couplers of 33 : 33 : 33 splitting ratios is used for multipath interferometry in the followed experiments. The three paths MZI was build using two cascaded symmetric tritters couplers. Just as in the unbalanced Mach-Zehnder \( (2 \times 2) \) interferometer, if there are loop delays \( \Delta T \) and \( 2\Delta T \) in two arms of the interferometer, any a train of pulses propagate inside the interferometer produces output interference independence of the choices of the phase modulation.

Devices with three arms inputs and outputs can build qutrits interferometer. The output states \( E_{out} \equiv (E_{1\prime}^{out} + E_{2\prime}^{out} + E_{3\prime}^{out}) \) from the devices can be represented if the input state named \( E_{in} \equiv (E_{1}^{in} + 0 + 0) \) using the transformation matrix (T):

\[
T = \frac{1}{\sqrt{3}} \begin{bmatrix}
1 & 1 & 1 \\
1 & e^{-i\frac{2\pi}{3}} & e^{i\frac{2\pi}{3}} \\
1 & e^{i\frac{2\pi}{3}} & e^{-i\frac{2\pi}{3}}
\end{bmatrix}.
\] (3.15)

Having phase modulations in two arms of the three elements, the electric field amplitudes outputs can be found using the phase modulation matrix (S) as:

\[
E_{out} = (TST) E_{in},
\] (3.16)
where $S$, the phase modulation matrix is:

$$
S = \frac{1}{\sqrt{3}} \begin{bmatrix}
1 & 0 & 0 \\
0 & e^{i\phi_2} & 0 \\
0 & 0 & e^{i\phi_3}
\end{bmatrix}.
$$

We are interested in intensities (the squared modulus of electric field) which can be written as:

$$
I_1' = \left(\frac{E_{in}^1}{9}\right)^2 \left\{3 + 2 \left[ \cos \phi_2 + \cos \phi_3 + \cos(\phi_2 - \phi_3) \right] \right\},
$$

$$
I_2' = \left(\frac{E_{in}^1}{9}\right)^2 \left\{3 + 2 \left[ \cos \left(\phi_2 - \frac{2\pi}{3}\right) + \cos \left(\phi_3 + \frac{2\pi}{3}\right) + \cos \left(\phi_2 - \phi_3 + \frac{2\pi}{3}\right) \right] \right\},
$$

$$
I_3' = \left(\frac{E_{in}^1}{9}\right)^2 \left\{3 + 2 \left[ \cos \left(\phi_2 + \frac{2\pi}{3}\right) + \cos \left(\phi_3 - \frac{2\pi}{3}\right) + \cos \left(\phi_2 - \phi_3 - \frac{2\pi}{3}\right) \right] \right\}.
$$

The relative outputs intensities depend on the phase differences, having a maximum in one port implies no light from the others.

### 3.3.2 Plug and Play configuration

One of the biggest issues facing optical setups containing series of interferometers and optical components linked together are the fluctuation on the signal polarization and phase over distances, the idea to use configuration leads to self-compensation of polarization and phase fluctuations with a high degree of stability over hours can be used as a solution to this problem, e.g., Plug and play configuration. Plug and play configuration is an interferometer based on Faraday mirror (An ordinary mirror, glued on a Faraday rotator (FR) to rotates the polarization by $45^\circ$, the effect of the mirror is to transform any polarization state into its orthogonal [9]). The combination of the mirror and the rotator results in a $90^\circ$ total rotation with respect to the input polarization. The configuration usage is to ensure the height stability of the interferometer, by letting the pulses take the same spatial path all the way and backward. The Faraday mirror suppresses all birefringence effects and polarization dependent losses in the interferometers. To give an illustration of the interferometers work, a schematic of the plug and play setup is presented in Fig.3.7 [10], a light source is used to send a pulse through the circulator CIR to the 50:50 beam splitter. The splitting pulses were transmitted through the short and the long arms (the long arm contains a delay line DL and a phase modulator $\phi_A$). The phase modulator $\phi_A$ kept inactive during the pulse propagation over the long arm.
Chapter 3. Optical fibers system

Figure 3.7: Plug and play configuration setup [10].

Pulses pass through to a polarization beam splitter (PBS) and leave the interferometer using the same output port arm (i.e., the long arm pulse is vertically polarized and the short one horizontally one), both pulses are transmitted over SM fiber link to Bob's station. The pulses reflected back inside Bob station due to the Faraday mirror towards Alice's station. Both pulses pass the SM fiber link back across the PBS into the opposite arms (relative to the forward propagation), the long arm pulse then be modulated, and the two pulses arrived at the same time at the 50:50 BS, where the interference took place. Depending on the phase difference between them, e.g., modulation by \( \phi_A \) on its backward direction to the 50:50 BS, results to an event to be detected either in the detector \( DET_1 \), if the phase difference \( \phi = \pi \pm 2n\pi \); or in both detectors for other \( \pi \) values, or in \( DET_2 \), if \( \phi = 0 \pm 2n\pi \), where \( n \) is an integer number.
4. Hacking Franson interferometer

Using tailored pulses of classical light, we attacked energy-time entangled system, by controlling the avalanche photo-detectors used in the experiment and reach Bell values up to 3.6 at 97.6% faked detector efficiency, which exceeds even the quantum limit [Paper I].

4.1 Introduction

Photonics systems based on energy-time entanglement have been used to test local realism using the Bell inequality. A violation of this inequality normally certifies the security of quantum key distribution (QKD), so that an attacker cannot eavesdrop or control the system. Here we show how this security can be circumvented in energy-time entangled systems when using standard avalanche photo-detectors APDs, allowing an attacker to compromise the system without leaving a trace.

4.2 Quantum cryptography (QC)

The idea of cryptography is to ensure the security when a message is transmitted over untrusted channel. One of the basic examples of system using cryptography is a two parties system, namely Alice and Bob wish to communicate in presence of eavesdroppers. Alice encrypts her message (plain text) using pre-determined algorithm using a key of encryption and her message become a ciphertext, that can be transmitted over a channel containing eavesdroppers, which they can assumed to have access to transmitted ciphertext message. Then Bob received the ciphertext message and decrypts it using the decryption key. If successful operation done the message will be recover. Moving to the quantum cryptography, Shor [11] published a quantum algorithm for prime factorization in polynomial time which will compromise the existing encryptions protocols. Nowadays several system using qubits in cryptogra-
phy are practically secured, e.g., the one-time pad. In quantum cryptography, a quantum channel used to allows Alice and Bob to set up a communication system, the laws of physics guarantee the security (the not vague concept of computational complexity). This channel guarantees that one can use a key in the one-time pad, a method where a quantum key distribution (QKD) in perfect security. Several protocols have been proposed in quantum key distribution, e.g., BB84 protocol by Bennett and Brassard [12], E91 by Ekert [13]. Later what called energy-time entanglement suggested by Franson [14] has been applied to quantum cryptography to achieve unconditional security and reported in many publication using it [15–21].

4.2.1 Quantum key distribution

QKD is a method involving quantum mechanics to perform quantum cryptography protocols in order to implement a secure communication (any eavesdropper presence will be detected). QKD has a unique property that any two communicating users can detect a third party trying to get information about the key.

4.2.2 BB84 protocol

The BB84 protocol is the first quantum cryptography protocol. It was proposed by Bennett and Brassard in 1984, which it has been named "BB84" [12]. The protocol relies on quantum mechanics, in which the users (e.g., Alice and Bob) are connected through a quantum channel (e.g., optical fiber using one photon per bit) and a classical public channel (e.g., internet connection [8]). Fig. 4.1 shows the protocol illustration [22]. The following steps give the principles of the protocol [23]:

Figure 4.1: Schematic illustration for the BB84 protocol reprinted from [23].
4.2. Quantum cryptography (QC)

- Alice chooses among four non-orthogonal states, i.e., Horizontal, Vertical, Diagonal and Anti-diagonal bases to encode the transmit information. If in case of setting the Horizontal and Diagonal for 0, while Vertical and Anti-diagonal for 1.

- Alice chooses randomly one of the state polarization for each photon and sends the corresponding state to Bob.

- Bob then performs measurements of the incoming state in one of the two bases, either Alice and Bob use the same basis (correlated results), or their measurement performed on a different basis (uncorrelated results). No information to Bob about the sent photon state. At sometimes because of the errors in the transmission or detection Bob was not be able to register anything.

- Bob then obtains a string of all received bits. Through the public channel he announces which basis was used for each photon that was registered without revealing the results has obtained.

- Later Alice and Bob compared their selected bases, they keep only the bits corresponding to the same basis. The process discarded 50% of the raw key.

- A set of bits were used by Alice and Bob for errors rate check. They run an errors correction methods to remove the errors.

- Alice and Bob perform the privacy amplification to remove any leaked information in the above steps.

The secret key in the protocol is truly random (neither Alice nor Bob decided it results because of random chose between the bases in the process).

**Eavesdropping**

Eve’s would like to get as much as possible information without leaving a trace detect her presence. One of the scenario for her is to use what call intercepts-resend strategy (of a qubit) Eve will use in her attack the same equipment as the one of Bob station. Eve can only chooses the bases randomly and in 50% of the cases she will guess the correct basis and resend a qubit in the correct state to Bob measurement station (while she has no information in the rest wrong bases). The amount of information Eve will get in her attack (the 50%) will let Alice and Bob obtain 25% error rate which reveals the presence of Eve. If Eve only chooses to apply the attack for a few percentage fractions of the
measurements (Eve would like to reduce the error rate) Alice and Bob can use classical algorithms (to correct the error rate first) and reduce Eve’s knowledge of the final key by privacy amplification [23].

4.2.3 E91 protocol

The E91 protocol, an approach for quantum key distribution using bell theorem to reveal and test the eavesdropping. Artur K. Ekert suggested in 1991 the protocol named "E91". Albert Einstein, Boris Podolsky and Nathan Rosen published a paper in 1935 where they (Known as EPR) asked if quantum mechanics could be considered complete [24]. EPR paper gives two assumptions, namely “locality” and “realism” as a requirement to complete the theory. In 1964, John Stewart Bell published a paper [1] driving famous results from the same assumption of EPR paper. Bell showed that the locality principle was not consistent with the local hidden variable theory as proposed by EPR. Bell in his theory provides Bell’s inequalities. E91 protocol is connecting to the EPR paradox. The steps below show the protocol work principle [13]:

- The protocol can be run using; qubits in a maximally entangled state like EPR and two measurements stations for Alice and Bob as in Fig. 4.2.

- Alice and Bob are randomly choose between three bases, obtained by rotating the horizontal-vertical basis, around the z-axis by angles. Alice orient her station along one of the angels $\phi_{A1} = 0$, $\phi_{A2} = \frac{1}{2}\pi$ and $\phi_{A3} = \frac{1}{2}\pi$, while Bob chose between $\phi_{B1} = \frac{1}{4}\pi$, $\phi_{B2} = \frac{1}{2}\pi$ and $\phi_{B3} = \frac{3}{4}\pi$. A measurement gives an output result $\pm 1$, (depending on whether the photons are measured at the first or second polarization state of the chosen basis).

- After performing a number of measurements, Alice and Bob publicly release which basis they have chosen for each measurement, in three
categories, i.e., different orientation group (use in bell’s test), same orientation group (use to establish a security key) and a group which at least one of them failed to registered a particle (to be discarded).

- Later Alice and Bob announce publicly the first group result and check if there any eavesdropping attack (i.e., violated CHSH inequality), otherwise they use the second group for their security key.

Eavesdropping

The eavesdropping Eve can attack the E91 protocols for example by replacing the entanglement source with a Trojan device.

4.3 Energy-time entanglement

Entanglement by means of polarization (as in E91 protocol) is a robust candidate for QKD. But crucial aspects over long distance fiber communication uncontrolled conditions environment, in one hand a high rise of communication protocol error occurs, and on the other hand, the maintaining cost of the signal through the channel rise up too. Recently a great interest in using non-commuting observables of energy and time instead of the polarization states to bring protocols that are not only applicable in a laboratory but robust over long distances in hard environment condition.

4.4 Franson interferometer

A Franson interferometer is a device that consists of two composed Mach-Zehnder interferometers. It was proposed for the Bell test experiment using unbalanced Mach-Zehnder interferometers as analysis stations. The general description of the setup used in the experiment is a central source connected to two measurement sites, named Alice and Bob. The source prepares an entangled quantum state and distributes it to Alice and Bob analysis stations, where each can then choose between numbers of measurement settings (using the phase modulator $\phi$) for their devices. The path-length difference, between time delays loops in both interferometers are equals and define as:

$$\triangle l = d_{long} - d_{short}.$$  \hspace{1cm} (4.1)
Chapter 4. Hacking Franson interferometer

Figure 4.3: Franson interferometer. The source emits time-correlated photons, measurements performs along angles $\phi_A$ and $\phi_B$ in Alice and Bob stations respectively.

This corresponds to transmit-time difference between the two paths in both interferometers define as:

$$\Delta T = \frac{\Delta l}{c}.$$  \hfill (4.2)

Alice and Bob record the time of the arrival photon give detection outcome measurement of $+1$ or $-1$ at the measurement analysis station. There will be quantum interference if the detection times at Alice’s and Bob’s detectors are equal, in the two following causes:

- The early source emission (when the photons are delayed at measurement stations)
- The late source emission (when the photons have no delayed at the measurement stations)

By repeating the process numbers of times, Alice and Bob correlation between the post-selected (coincidence) measurements outcomes as:

$$E(A(\phi_A)B(\phi_B)) = \cos(\phi_A + \phi_B).$$  \hfill (4.3)

Alice and Bob later announcing their post-selected measurement Alice setting $\phi_A$ and Bob one $\phi_B$ and compute the correlation in equation 4.3 and test the Bell CHSH inequality see section 2.1.6. The correlation coefficients then can be defined to compute the quantity $S$ (the Bell value for the E91 protocol) by Alice and Bob as:

$$S = |E(A_1B_1) + E(A_1B_2)| + |E(A_2B_1) - E(A_2B_2)| = 2\sqrt{2}. \hfill (4.4)$$

If no violation of CHSH inequality, an attack might attempt by Eve and Alice and Bob must stop the communication, in other words, Bell test is a security
test needed to pass in order to trust the key. In Franson interferometer for two setting per observable the outcomes are bounded by $\leq 4$, see [25] for more information.

### 4.5 Hacking quantum system

#### 4.5.1 Blinding the detectors

APDs in Geiger mode operation can sign a click, whenever an absorbed single photon resulting in a large current ($I_{APD}$) inside the APD’s circuit exceeds the comparator threshold and announce the photon’s presence [26]. APDs spend some time being biased under the reverse-biased, i.e., in the linear mode [27] (the detectors are sensitive only for classical light). By dropping the bias voltage in such case that the detector never works in the gated mode but the linear mode [28], the detectors are then fully controlled by using the CW illumination. The process causes a current lead to decrease the bias voltage across the APD, this current is proportional to the incoming light, here the threshold current determined the optical power threshold ($P$), meaning that the detectors keep silent unless a CW illumination above this threshold hit (then it will case a click). The process thus results in the transition of any gated detector from Geiger mode to a linear photo-diode mode that no longer be sensitive to single photons. In other words, the process converts the target detector to work as a classical linear detector instead.

The classical optical power threshold ($P$) has to be sufficiently well defined

![Figure 4.4](image)

**Figure 4.4:** In the Geiger mode, the APD is reverse-biased above the breakdown voltage $V_{br}$. A so-called a “click” occurs when $I_{APD}$ crosses the threshold $I_{th}$. Then $V_{APD}$ is lowered below $V_{br}$ to quench the avalanche. Before the APD returns to Geiger mode, in the linear mode (below $V_{br}$), the current $I_{APD}$ is proportional to the incident optical power $P_{opt}$, the current $I_{th}$ becomes determined an optical power threshold $P_{th}$ [28].
Figure 4.5: While the APDs are in the linear mode, (a) the bright pulse intensity is slightly above the optical power thresholds resulting a click at detector "0", or (b) the bright pulse has low-intensity optical power than the thresholds resulting no click events in the detectors [28].

for successful perfect eavesdropping as in the condition [27]:

\[ P(\text{max})_{\text{click}} < 2 P(\text{min})_{\text{no\text{-}click}} \] (4.5)

If APDs are used in a QKD system, Eve can optically get access by letting the APDs biased under the breakdown voltage, and may attack the QKD system with so-called intercept-resend (faked-state) attack. Here Eve can make a copy of Bob station (interferometer) to detect the qubits from Alice on a random basis. Eve resends her detection results, but instead of sending single photons she sends bright pulses, just above the classical optical power threshold. Bob will only have a detection event if his basis choice coincides with Eve’s basis choice, otherwise no detector clicks. In addition to attacking the quantum channel, Eve listens on the classical channel between Alice and Bob. Afterward, Eve performs the same classical post-processing as Bob to obtain the identical secret key.

4.5.2 Hacking scenario

Local hidden variable (LHV) model in Franson interferometer [25], is extended to have a discretization for LHV that gives the entire set of quantum predictions, by choosing the setting that needs to be used for the present Bell test. Random outcomes at the analysis stations of Alice and Bob can be performed by repeating the procedure that leads to Quantum predictions violating Bell CHSH inequality, while in reality using these random setting for the attack see the paper (I) for more information. Alice and Bob perform all their
4.5. Hacking quantum system measurements using the analysis stations and have exactly their expected statistical distributions. Our hacking scenario in one hand, the model prescribes the sign of the Alice and Bob measurement outcome \((\pm 1)\) and the arrival time (Early or Late), on the other hand, by blinding the detectors, the APDs will not distinguish between different quantum states of incoming light, no matter of the quantum properties of that pulse [29].

Let us now briefly go through the attack; the hacking process is done by constructing a bright pulse (by Eve) and used it to force a click on a target detector, i.e., a bright single pulse with an intensity of length \(\tau \ll \Delta T\) simultaneously with a CW light sufficient to blinds the detectors. Any bright single pulse of intensity will be divided inside the interferometer analysis stations resulting in two pulses of \((\frac{1}{4}I)\) in the outputs.

An accurate train of two pulses with time separation \(\Delta T\), emitted from the source, with phase shift \(\omega\) produced an outcome at the analysis stations consisting of three pulses where the middle one was composed by two parts. This output results in interference at the station detectors \((\pm 1)\) having intensities as

\[\begin{align*}
I_+ & = I_E + I_C \\
I_- & = I_E - I_C \\
I_0 & = I_C
\end{align*}\]

\[\begin{align*}
I_E & = I + \frac{1}{4}I \\
I_C & = I - \frac{1}{4}I \\
I & = \text{intensity of the bright pulse}
\]

\(\frac{1}{4}I\) in the outputs.

**Figure 4.6:** Constructive interference at the + output gives a large early time slot intensity and a corresponding click. Destructive interference at the - output gives a small early time slot intensity and no corresponding click.
follows:

\[ I_+(\phi, \omega) = I \cos^2 \left( \frac{\phi + \omega}{2} \right), \] (4.6)

\[ I_-(\phi, \omega) = I \sin^2 \left( \frac{\phi + \omega}{2} \right), \] (4.7)

where \( \phi \) is the phase setting of Franson interferometers analysis stations, which can be controlled in dependence of the output by choosing \( \omega \), e.g., for \( I < 2I_T \) in case of \( \omega = 0 \) and \( |\phi| < \frac{\pi}{2} \), this setting will give a click of +1, otherwise the click will be at −1. However a number of settings can be performed, by sending attack pulses to allow violation of Bell test, i.e., a train of three pulses from the fake state station generates with separated \( \Delta T \) time delay having a phase difference between the first and the second pulse by \( \omega_E \), and between the second and the third a phase different of \( \omega_L \). The output of the interference between the trains of pulses passing through, consists of a four output pulses, the middle two center pulses (early E, and late L) can be controlled by the interference, which has intensities as followed [25]:

\[ I_{+E}(\phi, \omega_E) = I \cos^2 \left( \frac{\phi + \omega_E}{2} \right), \] (4.8)

\[ I_{-E}(\phi, \omega_E) = I \sin^2 \left( \frac{\phi + \omega_E}{2} \right), \] (4.9)

\[ I_{+L}(\phi, \omega_L) = I \cos^2 \left( \frac{\phi + \omega_L}{2} \right), \] (4.10)

\[ I_{-L}(\phi, \omega_L) = I \sin^2 \left( \frac{\phi + \omega_L}{2} \right), \] (4.11)

A well prepared amplitude light pulses, as mention previously is used so that at the target detector with the right phase different and time slot, will forces a click as a result of a constructive interference, while no click will be noted if a destructive interference is present.

4.6 Experiment setup

The experiment was built using standard fiber-optical components and was designed to meet the requirements to produce fake states used to violate Bell inequality. The setup consists of two main parts, a fake light source and the two analysis stations can be seen in Fig.4.7 The fake light station consists of two types of light sources, i.e., a CW laser that produces continuous wave while a pulsed laser creates the pulses. These two light sources are combined at a \( 2 \times 2 \) fiber-optic coupler and then split into two arms, one beam arm was sent to Alice and the other to Bob, each through a \( 3 \times 3 \) fiber-optic coupler (triter)
4.7 Results and discussion

Figure 4.7: Attack experimental setup.

to equally divide the light into three arms. One arm consists of a $\Delta T$ delay loop and a phase modulator $\omega_E$, the second arm has two $\Delta T$ delay loops and a phase modulator $\omega_M$ (so that $\omega_L = \omega_M - \omega_E$) while the third arm performs no action. The three arms are then combined by a second $3 \times 3$ fiber optic-coupler, using one output port as the faked state source generator. This source sends bright light pulses with the setting and phase difference(s), i.e., the fake state, described in the previous section to Alice’s and Bob’s analysis stations, a two unbalance Mach-Zehnder in the Franson interferometer and performs the attack, see Fig.4.7.

4.7 Results and discussion

In this experiment, the APDs have a maximum detection efficiency of 20% and dark count of $5 \times 10^{-5} \text{ns}^{-1}$ at the operating temperature 218 K. The sampling time for the experiment was 1s, and each experiment setting was run for about 30 s. The APDs were attacked using bright classical pulses of light, described previously, with overlaid pulses were chosen from the LHV model. Detectors counts were used to computing the joint probabilities of Alice’s and Bob’s outcomes at each point of time, and then the results were used to determine the Bell value. In the attack, the Franson interferometer analysis stations measure a value that clearly violates Bell bound. The violation was $2.5615 \pm 0.0064$, has been shown by experimentally sampling time of 27s. The detectors under attack have a faked efficiency of 100% using the blinding process stated above, see Fig.4.8.

Bell inequality in the Franson interferometer is trivial, so one should be
Figure 4.8: The faked Bell value of our source is 2.561 (solid black line) which clearly violates the CHSH inequality. It is possible to increase the faked Bell value up to 3.639 (dotted blue line). In both cases, the faked efficiency is 97.6. Each point in the diagram corresponds to the $S_2$ value for 1s worth of data.

able to produce any Bell value between 0 and 4, but the end Bell value that can achieve which $3.6386 \pm 0.0096$ (the attackers will not prefer this value, for not to let Alice and Bob suspicious). Using the scenario, Alice and Bob can be fool by Eve, thinking their system violates Bell’s inequality, poorly they do not know that Eve blinds the detectors and uses an LHV model instead with no entanglement. In other words, Eve breaks the security of the Franson system without Alice or Bob noticing.

Avoiding the attack:

One of the suggested solutions is to replace the first beam splitter of the analysis station with a movable mirror [30–32], setup does not require post-selection at all, but still CHSH inequality applicable. A second solution, is by eliminating the post-selection loophole, e.g., use of “hugging” interferometers without post-selection [29; 30]. Another solution is to build a device-independent QKD system based on energy-time-entanglement having a fast switching plus replace the CHSH inequality with other stronger tests, e.g., modified Pearle-Braunstein-Caves inequalities.
4.8 Summary and Conclusion

In this work, it has been shown that quantum key distribution systems based on energy-time entanglement with post-selection are a matter to attack if the corresponding security tests use the original Bell inequality just by blinding the detectors and using an LHV model. The hacking attack was experimentally verified using passive fiber optics components. The circumvents even-though the security test based on Bell’s inequality, allow eavesdroppers to crack the key and read the message without detection.
5. Quantum multi-party communication

Experimentally, we demonstrated single qudit protocols constituting realizations of secret sharing, detectable Byzantine Agreement, clock synchronization, and reduction of communication complexity in a multi-partite setting involving three parties, Alice, Bob, and Charlie, communicating three-level quantum states (qutrits). We have realized three communication tasks with the same optical setup [Paper II].

5.1 Introduction

Quantum technologies often rely on strongly correlated data that could be distributed and cannot be reproduced using classical theory, e.g., violation of Bell inequality [1], where the parties involved must share an entangled quantum state and do suitable local measurements while its outcomes can locally be processed and communicated by classical means. Such entanglement-assisted protocols have been successfully reported in a variety of information processing tasks, e.g., including secret sharing, detectable Byzantine Agreement, and reduction of communication complexity [33–41]. In this chapter single qudit communication will be introduced, these protocols can be used as a quantum resource, in contrast to established entanglement-assisted one. Also in this chapter the demonstration of three-party quantum communication protocols using single qutrit communication: secret sharing, detectable Byzantine agreement, and communication complexity reduction for a three-valued function (Paper II) will be presented.

5.2 Qutrit

Given two qutrit (three level quantum state) vectors |k⟩ and |l⟩ from mutually unbiased bases, the following holds:

\[ |\langle k|l \rangle|^2 = \frac{1}{3}. \] (5.1)
Chapter 5. Quantum multi-party communication

The general qutrit state can be written as:

\[ |j\rangle = \frac{1}{\sqrt{3}} \sum_{k=0,1,2} e^{i(\omega jk + \phi_k)} |k\rangle, \tag{5.2} \]

where \( \omega = \frac{2\pi}{3} \), and we shall call this the \( \phi \)-representation. Tritters in qutrit are modelled by the unitary transformation:

\[ S_{kj} = \frac{1}{\sqrt{3}} \sum_k e^{\omega jk} |k\rangle \langle j|, \tag{5.3} \]

the local phase shifts are described by unitary transformation \( e^{i\phi_k} |k\rangle \langle k| \). In this representation, instead of the usual \( \{-1, 0, 1\} \) eigenvalues of the spin 1 operator, we use \( \{\lambda_0, \lambda_1, \lambda_2\} \) where \( \lambda = e^{i(\frac{2\pi}{3})} = e^{i\omega} \).

5.3 Single qutrit communication protocols

Secure communication between three different parties can be achieved by sharing a three-dimensional quantum object across an optical fiber network, i.e., using a single quantum object known as a qutrit that can exist in one of three states. Most quantum protocols for communication and information processing exploit entanglement as the main resource. Recently, there is an increasing interest in protocols based on sequential interventions on a single quantum system. Those protocols exploit quantum superposition as a resource and do not require composite systems, consequently, they are relatively easy to implement.

Single qudit communication protocols have clear advantages in terms of feasibility and scalability compared to known entanglement-assisted protocols. The

\[ \text{Figure 5.1: Schematic illustration for a single-qutrit, three-party communication protocols.} \]
5.3. Single qutrit communication protocols

A protocol reported in [38] is an example that relies on a cycle property of the set of MUBs. In this work, we implemented three three-party protocols for quantum information processing. The three experiments were conducted using the same optical fiber interferometric setup. The three protocols were conducted using the following steps in common:

- The first party prepares a single three-level system in a quantum state,
- the second party applies a unitary transformation on it,
- and the third party performs the measurement.

To distribute correlated lists using single qutrits, one can use devices that are able to perform unitary transformations on a single three-level quantum system. In a scenario involving sequential communication of this quantum system between devices (or processes), e.g., \( P_1, P_2, \) and \( P_3 \), assuming \( P_1 \) can prepare a single three-level quantum system randomly among some bases as a uniform superposition of initial state:

\[
|\psi_i\rangle = \frac{1}{\sqrt{3}}(|0\rangle + \omega^{\phi_1^i}|1\rangle + \omega^{\phi_2^i}|2\rangle),
\] (5.4)

where \( \phi_1^i \in \{0, 1, 2\} \), the parties then act sequentially with a unitary phase shift on the same qutrit, then the last party \( P_3 \) has the possibility to perform measurements on the system with a final state can be written as:

\[
|\psi_f\rangle = \left(\prod_{j=0}^{2} U_j\right)|\psi_i\rangle,
\] (5.5)

where \( U_j \) is a phase shift unitary transformation. The final state in the equation above can be re-written as:

\[
|\psi_f\rangle = \frac{1}{\sqrt{3}}(|0\rangle + \omega^{\Sigma_{j=0}^2 \phi_1^j}|1\rangle + \omega^{\Sigma_{j=0}^2 \phi_2^j}|2\rangle),
\] (5.6)

where \( \phi_1^j \) and \( \phi_2^j \in \{0, 1, 2\} \). The final party will then perform a measurement \( X \), in the computational basis \( \{|x_0\rangle, |x_1\rangle, |x_2\rangle\} \), with probabilities outcome \( (l) \) can be found from:

\[
P(l) = \frac{1}{9} |1 + \omega^{-l}\Sigma_{j=0}^2 \phi_1^j + \omega^{-2l}\Sigma_{j=0}^2 \phi_2^j|^2.
\] (5.7)
Chapter 5. Quantum multi-party communication

The probability distribution \( P(l) \) can be written in expanded form as:

\[
P(l) = \frac{1}{9} \left[ 3 + 2\cos\left(\frac{2\pi}{3} \left( \sum_{j=0}^{2} \phi^1_j - l \right) \right) 
+ 2\cos\left(\frac{2\pi}{3} \left( \sum_{j=0}^{2} (\phi^1_j - \phi^2_j + l) \right) \right) 
+ 2\cos\left(\frac{2\pi}{3} \left( \sum_{j=0}^{2} \phi^2_j - 2l \right) \right) \right].
\] (5.8)

In order for \( |\psi_f\rangle \) to be an eigenstate of \( X \) it is required that:

\[
\sum_{j=0}^{2} (\phi^1_j + \phi^2_j) = 0 \mod 3 \iff \langle X \rangle \in \{1, \omega, \omega^2\}.
\] (5.9)

The secret sharing distributor needs to verify that the condition in 5.9 is fulfilled. Experimentally we realized the following three protocols: multi-party quantum secret-sharing, a quantum solution to the Detectable Byzantine Agreement (DBA), which can be used to achieve clock synchronization (that aims to offer the constituent parts of a distributed system with a common notion of time) in the presence of an arbitrary number of faulty processes using efficient classical means of communications, and communication complexity reduction for a three-valued function that can be used to accomplish a globally defined task.

5.3.1 Quantum secret sharing

Quantum Secret sharing (QSS) is a cryptographic scheme that splits a secret message into several shares, so that the message can be recovered only if a certain number of shares is made known by means of collaboration [33; 35; 37–41]. In QSS, the secret is distributed among the users, in a way that no single user alone can reconstruct it, but that together they can have full information. The classical solution to Secret Sharing relies on the limited computational power of eavesdroppers trying to decrypt a secret key, therefore the unconditional security provided by QKD schemes is appropriate in Secret Sharing too. Besides, these schemes can be subjected not only to eavesdropping attacks, but also to betrayal from parties within the scheme [41]). Experimental QSS has been reported using both entanglement-based and single particle protocols, with qubits [35].

In our realization of QSS in (Paper II) we performed the experiment using qutrits, with three sequential users involved (see Fig.5.1). Alice (the distribu-
5.3. Single qutrit communication protocols

The qutrit state is prepared as:

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle).$$  \hspace{1cm} (5.10)

Alice applies the action $U^{a_0}V^{a_1}$ on $|\psi\rangle$, according to input data $a_0$, and $a_1$, two pseudorandom independent numbers in the set $\{0, 1, 2\}$. Operators $U$ and $V$ are given by:

$$U = |0\rangle\langle 0| + e^{\frac{2\pi i}{3}}|1\rangle\langle 1| + e^{-\frac{2\pi i}{3}}|2\rangle\langle 2|, \hspace{1cm} (5.11)$$

$$V = |0\rangle\langle 0| + e^{\frac{2\pi i}{3}}|1\rangle\langle 1| + e^{-\frac{2\pi i}{3}}|2\rangle\langle 2|. \hspace{1cm} (5.12)$$

Then Alice sends the qutrit to Bob, who according to his input data $b_0$ and $b_1$, acts on the qutrit with operator $U^{b_0}V^{b_1}$ and sends the state to Charlie who acts on the qutrit with operator $U^{c_0}V^{c_1}$ where $c_0$ and $c_1$, are his input data. Finally, Charlie performs a measurement on the qutrit in Fourier basis, to obtain a trit outcome $m$.

In random order, the parties then announce their data $a_1$, $b_1$, and $c_1$, and if condition $a_1 + b_1 + c_1 = 0 \mod 3$, is verified, the round is treated as valid and equation $a_0 + b_0 + c_0 = 0 \mod 3$ produces the shared secret. However, if $a_1 + b_1 + c_1 \neq 0 \mod 3$, the qutrit is not in an eigenstate of the operator at the time of measurement. Thus, the outcome $m$ is random and the run is discarded. At this point all users should publicly announce $a_0$, $b_0$, and $c_0$, for a relevant number of runs and estimate the quantum trit error rate (QTER). Finally, to reconstruct the shared secret at least two users are required to collaborate [42].

5.3.2 Detectable Byzantine Agreement

The Byzantine Agreement protocol is a distributed computing protocol, named after a problem (the word Byzantine itself refers to a historical problem) formulated by Lamport, Shostak, and Pease in 1982 [43]. The protocol aims at developing algorithms that do not require any centralized control with the guarantee of always working correctly [36], Fig. 5.2 show the schematic for DBA protocol, which is unsolvable by classical means, but with quantum resources. The goal of the protocol in the qutrit system of three parties is to distribute three correlated lists ($l_A$, $l_B$ and $l_C$) only known by the list holder and not by any other one. All the list have same length $L$, having the property that if 0 (1) is at position $k$ in $l_A$ then it should be at the position $k$ for both $l_B$ and $l_C$, and if 2 is at the position $k$ in $l_A$ then 0 will be at position $k$ of one of the lists while 1 is at the position $k$ on the other. Moreover, the party A knows exactly which positions of the list $l_A$ and $l_B$ are perfectly correlated and which not (but then
A has no idea who has 1 and who has 2.
The three parties A (the commanding party), B and C can communicate with one another via classical channel by messengers only. They must decide upon a common plan of action either 0 or 1 (meaning, for instance, attack or retreat). The commanding party A decides on a plan and communicates this plan to the other parties B and C, i.e., a message \( m_{AB} \) with either 0 or 1 and list \( l_{AB} \), and a message \( m_{AC} \), either 0 or 1 and a list \( l_{AC} \) respectively. After that, B communicates the plan to C by sending a message \( m_{BC} \) and a list \( l_{BC} \), and C communicates the plan to B by sending a message \( m_{CB} \) and a list \( l_{CB} \). However, one of the parties (including A) might be a traitor, trying to keep the loyal parties from agreeing on a plan.

The DBA protocol reported in [44], for qutrits and 3 parties (or processes) used to perform the experiment in order to solve the DBA, goes as follows:

- The party A has a source of qutrit while C has a measurement device.
- The party A prepares a qutrit of initial state:

\[
\left| \psi_i \right> = \frac{1}{\sqrt{3}} \sum_{j=0}^{2} \left| j \right>
\]

(5.13)

- The party A randomly encodes his basis and labels his choice \( c_1 \). The party A then applies the following unitary transformation to the qutrit:

\[
U_{c_1} = \left| 0 \right> \left< 0 \right| + \sum_{j=1}^{2} \omega^{c_1} \left| j \right> \left< j \right|
\]

(5.14)

where \( \omega = e^{\frac{2\pi i}{3}} \). applying \( U_{c_1} \) introduces a phase shift of \( \frac{2\pi c_1}{3} \) from the interferometric point of view.
• The party $A$ chooses $n_1 \in \{0,1,2\}$. The unitary below is applied to encode $n_1$:

$$U_{n_1} = |0\rangle \langle 0| + \sum_{j=0}^{2} \omega^{jn_1} |k\rangle \langle k|$$  \hspace{1cm} (5.15)

The qutrit is then sent to $B$, which in same manner as $A$ chooses $c_2 \in \{0,1,2\}$ and applies the unitary transformation $U_{c_2}$. Randomly $B$ chooses $n_2$, from the set $\{0,1\}$.

• If $n_2 = 1$, $B$ applies $U(n_2 = 1)$ and sends the qudit to $C$ (note: if $n_2 = 0$ no action will be taken).

• The same procedure will be repeated independently by $C$ with random value $n_3$. Finally $C$ measures the state the state $|\psi_f\rangle$. If $C$ obtains $|\psi_i\rangle$ all basis choices are revealed, but not the encoded numbers. The processes do this in reverse order ($C$ to $A$) and if the sum of the basis choices modulo $m$ equals zero, then the run is treated as a valid distribution of the numbers $n_k$ at the same position in the private lists $l_k$ and satisfied the condition:

$$\sum_{k=1}^{3} n_k = 0 \text{ mod } 3 \Leftrightarrow \prod_{k=1}^{3} U(n_k) = 1$$  \hspace{1cm} (5.16)

Under the above condition [36; 44–47], the three parties now hold one of the datasets $(a_1,b_1,c_1) \in (0,0,0), (1,1,1), (2,1,0), (2,0,1)$ from which the DBA task can be solved (Paper II).

5.3.3 Communication complexity reduction

In communication complexity problems (CCPs), local computations and exchange of information are performed by separate parties in order to accomplish a globally defined task, which is impossible to solve single-handedly. As an example, the task could consist in maximizing the probability of successfully computing a function, when the input is distributed among some parties, and the amount of communication is restricted.

For $N$ parties with each given two $d$-level inputs $x_i, y_i \in \{0,\ldots,d-1\}$, the common task of the $N$ parties is for one party (party number $N$) to work out the correct value of the function:

$$T_N = \sum_{i=1}^{N} S_i \text{ mod } d^2,$$  \hspace{1cm} (5.17)
where \( S_i = dx_i + y_i \) a combination of the parties’ inputs. By imposing a constraint on the input data:

\[
\sum_{i=1}^{N} S_i = 0 \mod d,
\]

which is equivalent to the constrain of the secret sharing.

We employed a single qutrit protocol for reducing the communication complexity with three separate parties (Alice, Bob, and Charlie). The distributor supplies Alice, Bob and Charlie with two pseudo-random trits each, \((a_0, a_1), (b_0, b_1)\) and \((c_0, c_1)\). Each party’s pair can be mapped into an integer by defining \( S_x = 3x_0 + x_1 \in \{0, ..., 8\} \), where \( x \in \{a, b, c\} \). The distributor promises the parties that \( S_a + S_b + S_c = 0 \mod 3 \), and asks Charlie to guess the value of function \( T = (S_a + S_b + S_c \mod 9) / 3 \), given that only two qutrits may be communicated in total. First, the following state will be prepared:

\[
|\psi_i\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle).
\]

Then Alice applies: \( U^{S_a}_3 \), with U defined as in equation 5.11, and sends the qutrit to Bob, who applies \( U^{S_b}_3 \), before forwarding it to Charlie. Finally, after applying \( U^{S_c}_3 \), Charlie performs a measurement on the resulting state:

\[
|\psi_f\rangle = \frac{1}{\sqrt{3}}(|0\rangle + e^{2\pi i 3T/3} |1\rangle + e^{-2\pi i 3T/3} |2\rangle).
\]

The above state is an element of the Fourier basis, thus a measurement in this basis will result in the output of the correct value of function \( T \), with unit probability. While an error in computing the function is allowed, the parties try to compute the function \( T \) correctly with a high probability. The success probability, save for experimental errors, is 100% with our quantum protocol. However, the optimal classical protocol achieves only a success probability of \( \frac{7}{9} \approx 0.778 \), which is clearly inferior [34; 48; 49].

5.3.4 Quantum error rates

In quantum communication, the information is carried by photons over long fiber links. Single mode fibers have on average a damping power of 0.25 dB/km, and the rate of receiving information for example qubits, can be expressed as [23]:

\[
Rate_{raw} = q \mu f \eta_{det} \eta_{link},
\]

where \( q \) is a system specific coefficient, \( \mu \) is the photon mean number per light pulse, \( f \) is the laser pulsing frequency, \( \eta_{det} \) is the probability of the photon’s
5.4. Experimental setup

detection, \( \eta_{\text{link}} \) is the efficiency of the communication links. In quantum communication, the errors due to the transmission are given by the quantum error rate. The quantum bit error rate (QBER) is calculated as [23]:

\[
QBER = QBER_{\text{opt}} + QBER_{\text{det}} = \frac{p_{\text{opt}} + p_{\text{noise}}}{p_{\text{photon}}},
\]

(5.22)

where \( p_{\text{opt}} \) is the probability of a photon hitting the wrong detector, and \( p_{\text{noise}} \) probability of the noise counts (which mainly from the dark counts) per gating pulse window. For the phase-based QKD the QBER can be calculated from the interference visibility as [50]:

\[
QBER = QBER_{\text{opt}} + QBER_{\text{det}} = \frac{1 - V}{2} \frac{p_{\text{noise}}}{p_{\text{photon}}}.
\]

(5.23)

For qutrit system, the quantum trit error rate (QTER) can be calculated as [40]:

\[
QTER = \frac{\text{Numer of incorrect outcomes}}{\text{Total number of outcomes}}.
\]

(5.24)

5.4 Experimental setup

We demonstrated three different protocols, i.e., secret sharing, detectable Byzantine agreement and communication complexity reduction (see the schematic illustration for the single qutrit parties communication protocol Fig. 5.1). We have implemented these three schemes using the same optical fiber interferometric see Fig.5.3. Our realization is easily scalable without sacrificing detection efficiency or generating extremely complex many-particle entangled states. The setup was built based on optical fibers: Alice station, which consists of a pulsed laser source, a Mach-Zehnder interferometer with two 3 \( \times \) 3 couplers (tritter), where the first tritter splits the coming pulse equally into three arms, two of these arms include delays \( \Delta L_L \) and \( \Delta L_M \). The signal then

![Figure 5.3: Qutrit communication protocols optical setup.](image)
is sent through a long SMF, link to Bob’s station which includes two phase-modulators, and again through another long SMF link to Charlie’s station, which consists of two phase-modulators in addition to a fiber pigtail mirror and an attenuator.

The information transmitted between users is encoded in relative phase differences between the three states constituting the qutrit. The state preparation is carried out by sending light pulses from a 1550 nm diode laser (ID300 by ID Quantique) to the first 3 × 3 coupler of the Mach-Zehnder interferometer. The laser repetition rate is 100 kHz. The outcome after the second coupler is a superposition of the three paths so that the optical phase of each pulse of the qutrit can be individually modulated with commercial phase modulators (COVEGA Mach-10 Lithium Niobate Modulators). The delays in the interferometer are \( \Delta L_M = 68.40 \pm 0.05 \) ns and \( \Delta L_L = 136.80 \pm 0.05 \) ns. On the way to the mirror, users passively let pass the qutrit through while after the reflection Charlie, Bob, and Alice sequentially act on the qutrit with a combination of operators \( U \) and \( V \) (see equations above). After passing through the three arms on their way back, the three pulses recombine at the first coupler and depending on their relative phases, yield different interference counts at the single photon detectors (Princeton Light-waves PGA 600 ). The output probabilities for the three outputs-arms of interferometer (see the Fig. 5.3) can be written as:

\[
P(D_0) = \frac{1}{9} \left\{ 3 + 2 \left[ \cos \phi_2 + \cos \phi_3 + \cos (\phi_2 - \phi_3) \right] \right\}, \tag{5.25}
\]

\[
P(D_1) = \frac{1}{9} \left\{ 3 + 2 \left[ \cos \left( \phi_2 - \frac{2\pi}{3} \right) + \cos \left( \phi_3 + \frac{2\pi}{3} \right) + \cos \left( \phi_2 - \phi_3 + \frac{2\pi}{3} \right) \right] \right\}, \tag{5.26}
\]

\[
P(D_2) = \frac{1}{9} \left\{ 3 + 2 \left[ \cos \left( \phi_2 + \frac{2\pi}{3} \right) + \cos \left( \phi_3 - \frac{2\pi}{3} \right) + \cos \left( \phi_2 - \phi_3 - \frac{2\pi}{3} \right) \right] \right\}. \tag{5.27}
\]

where \( \phi_2 \) and \( \phi_3 \) are phase differences relative to the reference arm.

The gated detectors used in the experiments shown 20% quantum efficiency and approximately \( 5 \times 10^5 \) dark counts probability. Importantly, in order to prevent possible eavesdropping attacks, each pulse is attenuated to single photon level by a digital variable attenuator (OZ Optics DA−100 ) at Charlie’s station output.

The phase modulators are polarization sensitive, and for this reason, they include a horizontal polarizer at the output port. Therefore, controlling polarization throughout the setup is crucial. We thus choose to use polarization maintaining fiber components for all three parties’ stations. However, in order to make the configuration more realistic, links between users are standard single
mode fibers. Therefore, polarization controllers have been placed after these fiber links. Finally, the whole experiment was controlled by an FPGA card that worked both as a master clock and as a trigger source, for the electronics-driving laser and phase modulators, and for the single photon detectors.

5.5 Results and discussion

Each of the protocols settings was run 100000 times per second (i.e. $10^5$ laser triggers) and the collected data was used to calculate the QTER. Due to the substantial loss from the setup itself (mainly in the phase modulators) and to the 20% detection efficiency, the final amount of runs with detection was about 400 per set.

We can see in Paper (II) tables 1 and 2, QTER for the secret sharing and DBA protocols are always below 10%. Our results are better than other results obtained with entanglement-based two-party quantum key distribution protocols (qutrits) [40], and QTER clearly are below the 15.95% [45] security threshold of qutrit based quantum key distribution. Therefore, secure communication can be obtained with this configuration [51]. Consistently, CCP experimental results, show success probabilities always above 90%, therefore, proving the superiority of the quantum protocol to any classical protocol (limited to 77.8% success probability). The primary source of QTER is the so-called “dark count”. Our detectors’ average dark count probabilities, measured with 1064 runs, are $5.9 \times 10^{-5}$, $2.8 \times 10^{-5}$, and $20.5 \times 10^{-5}$ per trigger for detectors 0, 1 and 2 respectively. Considering our measurements, these dark counts contribute up to half of the QTER. Other important systematic contributions to the QTER are due to the phase drift affecting the interferometer. This phase drift causes two problems: it slightly changes the relative phases from the desired settings and it forces a recalibration of the phases before each experiment. Both these contributions can be quantified, by propagating phase errors in interference equations, to be approximately 1% to each QTERs.

5.6 Summary and Conclusion

We have experimentally demonstrated three-party quantum communication protocols using single qutrit communication: secret sharing, detectable Byzantine agreement, and communication complexity reduction for a three-valued function. These three protocols have been implemented, for the first time, and we used the same optical fiber interferometric setup. Our novel protocols are based on single quantum system communication rather than entanglement. Moreover, the number of detectors (detector noise) in our schemes is indepen-
dent of the number of parties participating in the protocol. Our realization is
easily scalable without sacrificing detection efficiency or generating extremely
complex many-particle entangled states. These breakthrough and advances
make multi-party communication tasks feasible. They become technologically
comparable to quantum key distribution, so far the only commercial applica-
tion of quantum information. The realized and tested methods and techniques
can be generalized to other communication protocols. These protocols can be
easily adapted for other encodings and physical systems.
Part II

INTEGRATED OPTICAL CIRCUITS FOR QUANTUM INFORMATION
6. Waveguides technology using fs-laser writing

The second part of this work is dedicated to the possibilities of decreasing the dimensions of optical setups, thus carrying out experiments on chips rather than being table-sized. Such circuits offer a new level of robustness, mobility, stability, and compactness compared to more traditional optical setups. In following chapter I describe our study on writing waveguides in glass using fs-laser, focusing on designing, fabrication methods giving emphasis to experimental aspects in greater depth than in [paper III].

6.1 Introduction

The research work relies on the design, simulation and optimization of basic blocks, i.e., straight and curved line waveguides, directional couplers (DCs), Mach-Zehnder interferometers (MZIs) with $2 \times 2$ and $3 \times 3$ ports, as well as phase shifters. The miniaturization of these complicated optical systems can be achieved by writing optical circuits that exhibit the same physical effects inside of a glass sample as a bulk setup does in free space. The principal goal is to develop a method for design, fabrication, and characterization of integrated optics circuits for further applications in quantum information and integrated waveguides technology.

6.2 Integrated photonic circuits

An integrated photonic circuit is a device that enables the integration of multiple photonic functions. In contrast to electronic circuits, photonic circuits employ laser light at optical wavelengths, i.e., in the visible spectrum or near infra-red. Photonic integrated circuits have been fabricated in many substrate material, e.g., lithium niobate, silica on silicon, silicon on insulator, GaAs, InGaAs, and InP, as well as in various polymers. Many different techniques have been implemented to fabricate integrated photonics on chips. The most widely
used fabrication methods are lithography techniques and recently laser writing techniques.

6.2.1 Lithographic techniques

Among the fundamental methods to fabricate microstructured devices are photolithography techniques. Here the substrate surface is patterned with the structure one wants to transfer to the glass sample. The most basic scheme to fabricate waveguides using this technique involves following steps: first the sample substrate is coated with photore sist (PR) then the design pattern is transferred to the sample (through a mask containing circuit design) using UV light. In the second step, plasma etching forms the circuits in the glass material. Fig. 6.1 shows the photo-lithography steps of this simple scheme.

Figure 6.1: Fabrication steps of ridge waveguides in z-cut Lithium Niobate substrate, using a basic photo-lithography technique. (left) PR mask with an opening pattern on top of proton exchange lithium niobate layer PELN, to be etched. (middle) Plasma etching. (right) Ridge waveguides.

6.2.2 Fs-laser writing techniques

Another way for fabricating microstructured optical circuits is to write them directly into a substrate using focused laser pulses. The fabrication of optical waveguides by ultra-fast laser inscription relies extensively on non-linear energy deposition, close to the laser focus within a chosen glass substrate. Amplified femtosecond lasers provide an ideal source for the high peak powers necessary to drive these non-linear processes. Fs-laser pulses focused inside the bulk of transparent a material induce a local modification of the material’s refractive index and creates a guiding-medium for the propagating light. Direct fs-laser writing is particularly well suited for the integration of optical waveguides (or more complex photonics devices such as splitters and interferometers) and compared to traditional fabrication techniques, fs-laser waveguide writing offers a number of advantages:
6.2. Integrated photonic circuits

- No mask is required, making this fabrication technique rather fast.
- It is compatible with a wide range of materials.
- It can easily be used to fabricate buried optical waveguides in one single step.
- Optical circuits with three-dimensional layouts can be produced.
- Fs-laser writing can provide waveguides with a circular transverse profile and very low birefringence, thus supporting propagation of Gaussian modes with any polarization state. This makes it a particularly well suited method for rapid prototyping of devices.

In order to manufacture waveguides for high performance optical devices and certain wavelengths, careful optimization of following parameters is required [52–57]:

- Writing waveguides geometry,
- Pulse energy and repetition rate,
- Translation speed.

Two methods of writing can be distinguished, depending on the direction of writing translation:

- Longitudinal geometry, a method where the sample is translated along the fs laser beam propagation direction.
- Transverse geometry, a method where the sample is translated orthogonally to the fs laser beam propagation direction.

The latter was used in this work. However, in the case that the focusing beam propagates through the substrate at an angle (as it is the case when the laser beam impinges off-centre on the objective), the resulting waveguide has a tilted cross-section. This creates a waveguide with a rotated birefringence axis which can be used to fabricate integrated wave-plates [58].

Quantum optical experiments typically involve many discrete optical elements and thus suffer from complex alignment and instabilities caused by external perturbations. Integrated optical circuits have proven to be a valuable alternative for building robust and scalable quantum optics experiments.
6.2.3 The Femtosecond laser

The laser used for the inscription of optical samples is a fiber fs-laser, BlueCut from Menlo Systems. Based on Yb-doped fiber technology delivering femtosecond pulses with μJ power levels, linear polarization, and MHz repetition rates.\(^1\)

<table>
<thead>
<tr>
<th>Technical specification</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength, (\lambda)</td>
<td>1030 ±2 ([\text{nm}])</td>
</tr>
<tr>
<td>Pulse width, (\triangle T)</td>
<td>&lt; 500 ([\text{fs}])</td>
</tr>
<tr>
<td>Rep. Frequency, (f)</td>
<td>0.25-10 ([\text{MHz}])</td>
</tr>
<tr>
<td>Energy/pulse</td>
<td>&gt; 4 ([\mu\text{J}])</td>
</tr>
<tr>
<td>Power, (P)</td>
<td>&gt; 3 ([\text{W}])</td>
</tr>
<tr>
<td>Beam quality, (M^2)</td>
<td>&lt; 1.25</td>
</tr>
</tbody>
</table>

**Table 6.1:** Menlo System BlueCut Fs-laser specification.\(^1\)

6.2.4 Waveguides materials

Ultra-short laser pulses with pulse durations of hundreds of femtoseconds and pulse energies of hundreds of nJ can be focused in transparent materials and lead to non-linear light-matter interactions causing the formation of a free electron plasma due to the high peak intensity of the pulse (\(\sim 10 \text{ TW/cm}^2\)). This causes a permanent structural modification inside the material [54; 59; 60]. Since linear absorption phenomena are absent (the energy is smaller than the band gap in the transparent dielectric material) the pulse power is used solely for writing. The modification region can thus be controlled using an objective to realize micrometer scale spot sizes, while using 3D motion stages create a 3D structure. The extreme field intensity of the femtosecond laser pulses in the focus volume causes free electrons due to several effects [54]:

**Absorption of multiple photons:** The fs-laser pulses occur on time scales shorter than the energy transferred into the lattice, leading to free electron generation through multi-photon photo-ionization.

**Tunnelling ionisation:** The electric field of the fs-laser pulses reduce the valence and conductance band barrier and allow direct electron transition from the valence to conductance band.

**Avalanche ionisation:** An electron excited to the conduction band can continue to gain energy by direct photon absorption, a process which has a much higher probability than multi-photon absorption. The electron can gain enough energy through this process that it can excite another valence band electron to the conduction band which in turn can undergo the same process, creating an

\(^1\)BlueCut Femtosecond Fiber Laser. https://www.menlosystems.com/
6.3. Fs-laser writing setup

In order to perform waveguide inscription, additionally to a laser source capable of producing pulse energies sufficient to create smooth refractive index changes, the setup has to include a guiding and focusing objective and a translation stage capable of moving the glass sample through the focus with sufficient accuracy.

Figure 6.2: Comparison between (a) Linear absorption produced by CW light in a photosensitive material, and (b) Non-linear absorption produced by the pulsed laser in the transparent material.

avalanche effect an electron plasma until most of the pulse is absorbed. The kinetic energy gained by the electrons is transferred to the glass lattice as heat. The time scale of the avalanche ionisation process is on the order of 10 ps and thus 10 to 100 times longer than the pulse duration, while the thermal diffusion is in the order of 10 $\mu$s. By the end of the energy relaxation a permanent modification has occurred in the material [61–64]. How the energy absorbed introduces a refractive index change is not well understood and material dependent [65–67].

The glass I used for fabricating optical circuits is alumino-borosilicate Corning EAGLE 2000 $^1$ [68], which offers a low birefringence. The optical transmission window for this glass is shown in Fig 6.4.

Figure 6.3: Non-linear photo-ionisation in fs-laser machining. (a) Tunnelling ionisation, (b) multi-photon ionisation, and (c) avalanche carrier absorption followed by impact ionization. VB, valence band; CB, conduction band [54].

Figure 6.4: Transmission window of Corning Eagle2000 glass. At 1030 nm, the transmission of a 0.7 mm thick sample is roughly 90% [68].
6.3. Fs-laser writing setup

<table>
<thead>
<tr>
<th>Glass properties</th>
<th>Corning EAGLE2000</th>
<th>Corning 7980</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>Alkaline earth aluminoborosilicate</td>
<td>Fused silica</td>
</tr>
<tr>
<td>Density ( \text{g/cm}^3 )</td>
<td>2.37</td>
<td>2.20</td>
</tr>
<tr>
<td>Thermal diffusivity ( \text{cm}^2/\text{s} )</td>
<td>( 5.11 \times 10^{-3} )</td>
<td>( 7.50 \times 10^{-3} )</td>
</tr>
<tr>
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<td>1.456370</td>
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<td>Softening point</td>
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<td>1585(^\circ)</td>
</tr>
</tbody>
</table>

**Table 6.2:** Corning EAGLE2000 glass structural, thermal and optical properties at room temperature.

6.3.1 Guiding and focusing optics

The system for fabricating optical circuits in glass uses the already described femtosecond IR laser, and mirrors and pinholes for alignment. The femtosecond laser beam is focused inside (the focusing system influences the size and form of the waveguides,) the glass sample to form a localized refractive index change.

![Schematics of guiding and focusing optics.](image)

**Figure 6.5:** Schematics of guiding and focusing optics.

The BlueCut system emits laser light at a wavelength of 1030 nm, with the possibility to control the power used to write waveguides in borosilicate...
6.3.2 Translation stage and the 3D writing control

The translation stage A3200 consists of three air-bearing direct-drive linear translation stages\(^1\). X, Y, and Z-stage mounted together on a massive granite stone for mechanical stability, these stages incorporate non-contact air bearing surfaces. There is no mechanical contact to wear. Inherently frictionless, such a setup is standard for high precision applications.

![Aerotech 3200 translation stages forming the 3D-axis](https://www.aerotech.com/)

**Figure 6.6:** Aerotech 3200 translation stages forming the 3D-axis. X-direction (ABL 10150) 0-150 mm. Y-direction (ABL 10100) 0-100 mm. Z-direction (ABL 10025) 0-25 mm.

The substrate is translated with respect to the focus, thereby forming lines of index change which can be used as waveguides. The laser can be focused at multiple depths inside the sample meaning truly 3D waveguides circuits can be written.

6.4 Fabrication of optical circuits on chip

6.4.1 Waveguides and circuits design

A Python program is used to design the circuits. The Python program writes a text file, consisting of a table coordinates (x, y, and z), for the waveguides circuits which is called by the translation stage controller. Programs have been created for the following waveguides and circuits:

- Straight line waveguides.
- Curved line waveguides.

\(^1\)Aerotech 3200 translation stages. https://www.aerotech.com/
6.4. Fabrication of optical circuits on chip

- Directional coupler 2x2 (beam splitter).
- Directional coupler 3x3 (tritter).
- MZI with two cascaded 2x2 beam splitter and 3x3 tritters.
- Multi-ports circuits with several 2x2 beam splitters.
- Topological photonic lattices circuits in 2D (lines) and 3D (triangle and rhombus lattices).

6.4.2 Waveguides fabrication

All waveguides discussed in this thesis are fabricated in Corning EAGLE2000 borosilicate glass using the fs-laser beam. To study and optimize different building blocks for integrated photonic quantum circuits it is very important to fabricate three-dimensional low-loss integrated optical devices. The optimization for a waveguides recipe for guiding 780 nm light was done by writing a series of waveguides fabricated in a borosilicate sample with the beam focused at a depth of 170 µm below the sample surface. Several lines have been inscribed for different writing speed (10 mm/s, 20 mm/s, 30 mm/s, 40 mm/s, and 50 mm/s) while applying different pulse energy each time (200 nJ, 210 nJ, 220 nJ, 230 nJ, and 240 nJ). The samples were fabricated with a fixed laser output repetition rate and a 0.55 NA objective for focusing the beam inside the glass sample. The Waveguides were separated by 250 µm.

6.4.3 Sample preparation

Before characterizing them, the written waveguide samples have to undergo a few preparation steps. They are inspected with a microscope for unexpected defects in writing uniformity or due to the polishing process. The polishing of the sample facets is a very important step in the manufacturing of written optical circuit, because of connection losses with optical fibers, fiber arrays, etc. Well-polished facets give higher coupling efficiencies, fewer contact losses and better mode matching at connections. Optical facets of samples can be polished in two ways: manually or automatically with help of an available polishing machine\(^1\). Both methods result in a sample facet surface roughness of down to 0.1 µm. The first step of the polishing is done with a polishing paper of 35 µm roughness in order to to get correct flat surface in the facet, subsequent smaller grained polishing papers cannot correct the facet flatness any more. In order to get better contacts between inscribed waveguides and

---

\(^1\)Multiprep\(^\text{TM}\) Polishing system - 12", Allied high tech products.inc
In/Out optical fibers the surface roughness is decreased to about 0.1 µm. The end quality of the facets is checked with a microscope.

6.4.4 Waveguide characterization

The main configuration used for the waveguides characterization of the waveguide performance reported in this thesis work is shown in Fig. 6.8.

First microscopic inspection of the waveguides was done to investigate the waveguides widths as a function of writing speed and pulse energy, see the Fig. 6.9. The process is followed by optical characterization.

The characterization setup consists of a sample holder with 3 degrees of freedom. Light can be injected into the waveguides in two ways: the first method uses an objective (0.25 NA) mounted on a 5 freedom positioning stage holder, the second method uses input butt-coupling from a fiber grooves array, and an output collecting objective (0.6 NA) mounted on a 5 degrees of freedom positioning stage.

In the characterization process, the propagated output mode was measured as well as the Gaussian beam mode field diameter (MFD) which is important for the estimation of the coupling losses into and out of the waveguides. The MFD is also important for calculating the refractive index change and measurements of polarization maintenance.
6.4. Fabrication of optical circuits on chip

**Figure 6.9:** Waveguides width vs scanning speed.

**Figure 6.10:** Characterization of straight waveguides using butt-coupling. (a) Insertion loss characterization setup. (b) Setup for the measurements of 780 nm fiber mode profile and coupling used as a reference.

Waveguides output mode profile and efficiency:

The samples used in the study were excited by 780 nm CW laser \(^1\) and the output profiles were monitored using a CCD Camera \(^2\). Result for individual

\(^1\)Single Mode Diode Laser, Toptica Photonics
\(^2\)DCC1645C, Thorlabs
waveguides from each recipe is shown in Figure 6.11. The optimized parameters for waveguides supporting single mode 780 nm light are summarized in table 6.3.

![Figure 6.11: Output mode in dependence of writing translation speed and energy per pulse characterized using 780 nm light.](image)

Figure 6.12: The output mode from a waveguide fabricated using the recipe in table 6.3 excited using 780 nm light.

The waveguides mode profile $E(x,y)$ is an important characterization step when studying and optimizing the waveguides output. By assuming a mono-mode waveguide, the electric field profile $|\hat{E}(x,y)|$ can be written as:

$$|\hat{E}(x,y)| = C \sqrt{I(x,y)},$$

(6.1)

where $I(x,y)$ is spatial intensity profile and $C$ is a constant. Since we are interested in the normalised profile and since the electric field profile of the fundamental mode grantees the absence of any sign inversion in the electric...
6.4. Fabrication of optical circuits on chip

Fabrication parameters

<table>
<thead>
<tr>
<th>Fabrication parameters</th>
<th>Borosilicate</th>
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<td>Objective</td>
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</tr>
<tr>
<td>Pulse energy</td>
<td>210 nJ</td>
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</table>

Table 6.3: The optimized fabrication parameters for waveguides support 780 nm light wavelength.

field profile, C can be neglected. In order to characterize the mode size, we calibrate our measurement with the following three steps:

- First, the light was injected into the waveguide from a reference single mode fiber with known fiber mode size and NA and the output mode was imaged using the characterization setup in Fig. 6.10 (a).

- Second, the sample was removed, but without interfering with the projection objective and the beam profiler, to guarantee the same magnification. See Fig. 6.10 (b).

- At last, the fiber was brought close to the projection objective until the fiber mode image is found on the camera.

Since the fiber mode size and NA are known, it is possible to obtain the correspondence between the pixel size in the waveguide mode image and the real size of the mode, and also the waveguides NA. The waveguides support a single Gaussian mode of circular profile mode at 780 nm with a radius of 8 µm. The refractive index change for the waveguides written in borosilicate glass can be estimated by:

\[ NA = \sqrt{n_{wg}^2 - n_{glass}^2}, \]  

(6.2)

where \( n_{wg} \) and \( n_{glass} \) are the refractive indices of the waveguide and the glass substrate respectively. For waveguides with NA = 0.09, the refractive index change is \( 3.4 \times 10^{-3} \).

Waveguides birefringence:

The waveguides birefringence define as:

\[ \Delta n = \frac{\lambda}{2\pi \cdot L} \cdot \Delta \phi, \]  

(6.3)

where \( \Delta n \equiv \text{birefringence} \), \( \Delta \phi \equiv \text{the phase shift between the ordinary and extraordinary polarized wave found from the outputs tomography and L is the} \)
length of the waveguides. The setup in Fig. 6.13 is used for the birefringence characterization. The waveguides were excited using a 780 nm light source. The states $|H\rangle$, $|V\rangle$, $|L\rangle$, $|R\rangle$, $|+\rangle$ and $|-\rangle$ were prepared using the system to the left. The states are selected by rotating a $\lambda/2$ and if necessary a $\lambda/4$ waveplate. For each input state, the output state is projected on the same six polarization states by rotating the waveplates $\lambda/2$ and $\lambda/4$ on the right-hand side of the setup. The measurements are registered at the output port, and with help of these projections, we calculated the normalized Stokes vector for the output state. The waveguides birefringence value was measured to be $\Delta n = 7 \times 10^{-5}$.

**Figure 6.13:** Birefringence characterization setup.

with the fast axis to be aligned with TE polarization.

**Waveguide losses:**

The passive performance of a waveguide is described by the insertion loss (IL) which come from the contributions of coupling losses (CL), Fresnel losses (FL), and the propagation loss (PL). The insertion loss is due to:

- Input coupling losses when injecting light into the waveguides.
- Fresnel loss at input facet.
- The propagation losses along the waveguides.
- Fresnel loss at output facet.
- The output coupling loss for the signal collected from the waveguides.

Normally the losses described above are expressed in decibels (dB). The insertion loss is defined as the loss introduced to the signal when the waveguides are inserted in the path and take the form:

$$IL(dB) = CL_{in}(dB) + FL_{in}(dB) + PL(dB) + FL_{out}(dB) + CL_{out}(dB), \quad (6.4)$$
where:
\[ IL(dB) = 10 \log_{10} \left( \frac{P_{out}}{P_{in}} \right) \]  
(6.5)

for the case when the input power and output power of the device are measured. The optical coupling losses are defined as the losses introduced when the light is coupled from one optical medium to another. It depends on the integral overlap between the two modes of the first and the second medium.

\[ CL(dB) = -(10) \log_{10} (I_C), \]  
(6.6)

\[ I_C = \int \int \frac{E E dx dy}{(Edx dy)(Edx dy)}. \]  
(6.7)

Fresnel losses are defined as the losses from the glass interface Fresnel reflection:

\[ FL(dB) = -(10) \log_{10} (F_c), \]  
(6.8)

\[ F_c = 1 - \frac{(n_{glass} - n_{air})^2}{(n_{glass} + n_{air})^2}. \]  
(6.9)

**Loss measurement:**

The coupling losses are the losses from coupling light from the single-mode fiber to the waveguides and from the waveguides to an objective (or another fiber).

Mode field diameters of the input fiber and the output from the waveguides are measured with a beam profile.

By direct measurements of insertion losses and coupling losses, through equations [6.4] and [6.7], the losses for the glass can be estimated to be 0.03 dB/facet (including Fresnel losses). The propagation losses were calculated to be 0.3 dB/cm for the single mode waveguides at 780 nm wavelength.

**Bending curvature in waveguides:**

The losses in curves are a problem with waveguides taking a sharp turn. So-called S bend waveguides play an important role when computing losses. The quality of low-losses bend waveguides and their optimization are studied by fabricating a series of S-bend waveguides with a radius of curvature varying from 10 mm to 90 mm. The recipe for bend single mode waveguides supporting 780 nm wavelength is shown Table 6.3. The losses associated with each radius of curvature are shown in Fig. 6.14. The bending loss was measured and found to contribute with an additional 0.3 dB/cm in circuits with a 30 mm
radius and with less than 0.2 dB/cm when the waveguides were designed with 60 mm radius of curvature. A radius of curvatures higher that 50 mm was used to bring the circuits close to the surface to fabricate phase shifters for tunable devices see Chapter 7.

6.5 Summary and Conclusion

In this Chapter, an introduction to integrated photonic circuits, the methodologies for the fs-laser writing, the characterization and optimization of the optical circuit was given. This included an overview of the femtosecond laser writing system, guiding and focusing optics, and 3D motion control stages. Great care was taken on the optimization for waveguides and the characterization methods to assess the waveguides output intensity mode profiles, and optical losses were also presented.

We demonstrated optical Photonic waveguides structures in a chip, the realization showed guiding, confinements of the light. The waveguides with NA = 0.09 has a refractive index change of $3.4 \times 10^{-3}$ and its birefringence value was measured to be $7 \times 10^{-5}$. The scripted waveguides show 0.3 dB/cm propagation loss, an additional 0.2 dB/cm for the waveguides designed with 60 mm radius of curvature.
7. Fabrication of tunable integrated devices by fs-laser writing

In this chapter, I describe the methods I used to fabricate tunable integrated devices using fs-laser writing.

7.1 Introduction

Large-scale photonics architectures require that many components be connected with interferometric stability, and that the components show excellent performance stability. Integrated photonic circuits in chip show more potential for scaling up than table-top setups, because the component size is small and interferometric stability is inherent to integrated circuits [58; 69–76]. In this chapter key components (beam splitters, tunable phase shifters, directional couplers, 2×2 and 3×3 port Mach-Zehnder interferometers) are fabricated using fs-laser pulses. In integrated photonic devices for quantum information processing, a very important need is the reconfigurability of such devices. Reconfigurability allows compensation of fabrication errors, as well several quantum protocols require that the functionality of the circuit is changed dynamically.

7.2 Directional coupler

The directional coupler (DC) is equivalent to a bulk optics beam splitter. It has two input ports and two output ports.

DC design

The DC consists of two waveguides that are brought close to each other, separated by “close distance” d, for a length L, forming a region in which the
two waveguides interact with each other through evanescent field coupling. To bring the two waveguides to an interaction region one needs to curve the waveguides. An S-bend shape with a low (60 mm radius of curvature) bend loss is used. This guarantees symmetry in device fabrication. See Fig. 7.1. In analogy with a bulk optics device, one can define the power transmissivity (T) and reflectivity (R) as follows:

\[ T = \frac{P_{\text{cross}}}{P_{\text{cross}} + P_{\text{bar}}}, \]  
(7.1)

\[ R = \frac{P_{\text{bar}}}{P_{\text{cross}} + P_{\text{bar}}}, \]  
(7.2)

where \( P_{\text{cross}} \) and \( P_{\text{bar}} \) are the output powers of the device arms. The dependence of \( T \) on \( d \) is shown in Fig. 7.2.

Figure 7.1: Directional coupler design.

Figure 7.2: (Left) The DC transmission depends on the close distance (d). (Right) Experimentally measured coupling coefficients as a function of the waveguide separation (d) in the DCs.

dence of \( T \) on \( d \) is shown in Fig. 7.2.
7.2. Directional coupler

DC fabrication

The devices were fabricated using the Menlo BlueCut fs-laser in borosilicate glass (46 mm × 2.5 mm × 1.1 mm). The waveguides were fabricated by focusing the 1030 nm laser light 0.1 mm below the sample surface. The waveguides support one single mode at 780 nm. Parameters used for laser writing are shown in Table 7.1.

To produce compact circuits for quantum optics and quantum information experiments one needs to minimize the length of the directional coupler. Shorter DCs require higher coupling coefficients and hence smaller close distance $d$. However, reducing the distance between the waveguides comes at a price; the birefringence increases for closer waveguides and at some point, the two waveguides will overlap.

DC splitting ratio

The splitting ratio is tuned by varying the close distance $d$ and the length $L$. To study the splitting ratio a series of DCs were fabricated with different $L$ and $d$. In Fig. 7.2 we see the normalized transmission (Eq 7.1) oscillates as $L$ is varied. Assuming uniform losses in the waveguides, $T$ and $R$ oscillate:

$$T = \frac{k^2}{\sigma^2} \sin^2(\sigma L + \phi), \quad (7.3)$$

$$R = 1 - T = \frac{\Delta^2}{4\sigma^2} + \frac{k^2}{\sigma^2} \cos^2(\sigma L + \phi), \quad (7.4)$$

where $\sigma$ is the angular frequency, which is defined as $\sigma^2 = \kappa^2 + \frac{\Delta^2}{4}$, where $\kappa$ is the coupling coefficient between the optical modes, $\Delta$ is the optical modes detuning in propagation constant, $L$ is the interaction length, and $\phi$ taking into account the coupling occurring in the curved waveguides segments when two waveguides brought to a close distance $d$. By manipulating the interaction length $L$ and the separation $d$ of the directional coupler design waveguides...
circuit, one can fully tune the coupler splitting ratio. Using identical waveguides ($\Delta = 0$) the above equations can be written as:

\[ T = \sin^2(\kappa L + \phi) , \]  

\[ R = \cos^2(\kappa L + \phi) . \]  

\[ (7.5) \]

\[ (7.6) \]

**Figure 7.3:** Transmission H polarized light (Upper), Transmission V polarized light (Bottom), dependence on interaction length. The DCs fabricated using 240 nJ pulse energy, 40 mm/s scanning speed, 7µm separation distance, and 30 mm Radius of curvature.

The coupling coefficient depends on the overlap integral between the two waveguides, thus it depends on the separation distance $d$ between the two waveguides see Fig.7.2, it can be retrieved by inverting the relations 7.5 and 7.6.
Hong-Ou-Mandel effect observed in chip

DCs can be investigated at the single-photon level via a Hong-Ou-Mandel experiment. A DC was fabricated in a glass sample, a single photon was input into each of the input arms and the transmitted photons were monitored via detectors connected to each output arm. A light source which produced a pair of entangled photons, via spontaneous parametric down conversion, was used. The coincidence rate was monitored as the length of one of the input paths was varied; a quantum interference dip was observed with 96% visibility, see Fig. 7.4. The single mode fiber array was used to input the light into the chip and the multi mode fiber array was used to collect the light from the chip. Multi-mode fibers were used at the output for better collection efficiency.

Figure 7.4: Quantum interference Dip.

Figure 7.5: Directional Coupler in chip.
7.3 Phase shift control in a chip

To introduce a phase shift in a propagating mode resistive heating is used to change the temperature in part of the waveguide; because the refractive index is temperature-dependent a phase shift results. The phase shift is varied by changing the current through the resistor.

Resistors were made from 50 nm thick gold layers that were deposited on top of the waveguides. Two methods were used for their fabrication: photolithography and laser writing. In both methods a 3 nm thick layer of chromium was deposited before the gold layer, the chromium caused the gold to adhere to the glass substrate. Photo-lithography is described in Fig.7.8. The laser writing method started with deposition of chromium and gold over the entire substrate, then fs-laser writing was used to isolate resistive islands from the main gold layer. The laser writing one is fast and simple. A single laser writing scan removes gold layer in a line of width 2.5 \( \mu \text{m} \) with no damage to the glass substrate. The laser writing recipe is shown in Table 7.8. Surface references are important for alignment in the laser writing method. The resistance depends on \( \rho \) the resistivity of gold, \( l \) the fabricated resistance length and \( A \) the cross section:

\[
R = \rho \frac{l}{A}. \tag{7.7}
\]

To implement the phase shift the waveguides are brought close to the resistors.

**Figure 7.6:** (left) Resistor fabricated by photolithographic techniques. (right) Resistors fabricated using Fs laser writing.

**Figure 7.7:** Waveguides design, while raising one arm to meet the resistor on top of the glass sample, the other arm is kept immersed so that it does not experience a temperature shift.
7.3. Phase shift control in a chip

Figure 7.8: Fabrication of thermoresistive phase shifter using photolithography. (a) and (b) Photo-resist coating. (c) Pre-baking of the photo-resist. (d) UV exposure. (e) Development process. (f) Sample then has mask imprinted in photo-resist. (g) Gold coating. (h) Photo-resist removed, fabrication complete.

by raising them within the substrate to a depth of 25 µm.

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<tr>
<th>Fabrication parameters</th>
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<tbody>
<tr>
<td>Pulse repetition rate</td>
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<td>Objective</td>
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<td>Writing wavelength</td>
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<tr>
<td>Writing speed</td>
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<tr>
<td>Pulse energy</td>
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</table>

Table 7.2: Resistors pattern optimized recipe.
7.4 Tunable devices in chips

Having fabricated passive devices, by depositing and patterning gold resistors on top of the sample, the circuits become thermally reconfigurable, due to the refractive index change of waveguides underneath, which lead to phase accumulation between the interferometer arms.

7.4.1 Directional coupler

Directional couplers present strong potential in performing quantum information processing [69; 72; 74]. Here the devices are fabricated using two distinct waveguides (support 780 nm light) brought close together for 7 µm. The bending radius used in the design was a 60 mm. One arm is raised to a depth of 25 µm below the surface and the thermo-resistive phase shifter see Fig. 7.9. The laser writing recipe is shown in Table 7.3.

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</table>

**Table 7.3**: Tunable directional coupler waveguides circuit optimized recipe.

![Figure 7.9: DC beam splitter design.](image)

7.4.2 Mach-Zehnder Interferometer in chip

The Mach-Zehnder interferometer (MZI) is an optical circuit composed of two 2 × 2 beam splitters, and with a tunable phase introduced on one of the arms
between the beam splitters [69; 70; 75; 76]. A MZI is produced in chip using two DCs and a thermo-resistive phase shifter mounted on the chip surface above one of the arms (Fig. 7.10).

The MZI is inscribed at 100 µm depth below the sample surface. DCs with 1:1 splitting ratio are fabricated using 7 µm close distance. The waveguides are brought close together using a 60 mm bending radius. Away from the DC regime the waveguides are separated by 2 mm, each arm is a 7 mm waveguide long. Between the DCs one arm is raised to a depth of 25 µm below the surface and the thermo-resistive phase shifter, while the other arm is lowered to a depth of 175 µm – this ensures thermal isolation. After polishing the facets the sample has dimensions 46 mm long, 1.1 mm thick and 25 mm width. The device is compact and many such devices may be fabricated in a single sample. The device is fabricated in several steps: first the 3D waveguide circuit is written using fs-laser writing, and then gold resistors were fabricated using either photolithography or fs-laser writing (see section 7.3). Reference lines

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**Table 7.4:** Mach-Zehender interferometer waveguides circuit optimized recipe.

were also machined on the surface. The waveguides were fabricated in borosilicate glass using the Menlo BlueCut fs-laser with the parameters in Table 7.4. Multi-mode waveguides were used to allow better coupling of light into the waveguides.

![Figure 7.10: MZ interferometer device design.](image-url)
7.5 Results and discussion

The tunable integrated circuit components were first characterised using classical light. The refractive index change for the inscribed waveguides is $3.9 \times 10^{-3}$. The waveguides have birefringence $B \equiv \Delta n = 7 \times 10^{-5}$, with fast axis aligned with the TE polarisation. The propagation losses is $0.3 \text{ dB/cm}$. S-bends have additional losses $0.35 \text{ dB/cm}$. Moreover the cross talk between two straight waveguides separated by $250 \mu\text{m}$ is $0.001\%$. A phase delay is introduced by resistively heating the gold resistors. The response time of the phase shifter to a change in current is around $1.9 \text{ s}$, and thus the circuit settles within a few seconds.

![Figure 7.11: Phase shifter response time.](image)

The response of the fabricated $2 \times 2$ MZI was characterised as a function of the heat dissipated by the resistors, the results are shown in Fig. 7.12. The operation of the interferometers was monitored for 3 hours under constant driving conditions. No evidence of phase drifts was observed. Shorter interaction length is used in fabrication which is preferable for complex compact circuits see Figs. 7.3.
7.6. Outlook and Future work

Other circuits, i.e., Mach-Zehnder $3 \times 3$ and Multiport circuit for Qutrit-qutrit Bell test violation were designed and fabricated by building waveguide interferometers associated with thermo-resistive elements.

$3 \times 3$ Mach-Zehnder Interferometer in chip

MZI circuits with two cascaded 3x3 beam splitters were fabricated in chips. The circuits were written using fs-laser writing in borosilicate glass (Fig. 7.13). The outer two arms have tunable phases, this is implemented using thermo-resistive phase shifters located above the arms.

The waveguides were inscribed at 100 $\mu$m depth below the sample surface. 1:1:1 splitting ratios were obtained with a waveguide separation $d = 7 \mu$m. The bending radius used was again 60 mm. Between the cascaded DCs the waveguides were separated by 2 mm. The outer arms were raised to 25 $\mu$m depth near the phase shifts while the inner arm was lowered to 175 $\mu$m depth. The sample length is 46 mm and the width is 25 mm. The fabrication recipe is in Table 7.4.
Multiport circuits

Fs-laser writing is a powerful tool for constructing multiport circuits capable of realising complicated quantum tasks, such as boson sampling [72; 74; 77; 78]. Here a multi-port tunable device was constructed in borosilicate glass (Corning EAGLE2000) using the BlueCut fs-laser system. The device is shown in Fig. 7.14. The optimised recipe for producing the circuit with 780nm light

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<td>Pulse energy</td>
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Table 7.5: Directional coupler waveguides circuit (design with 30 mm radius curvature) optimized recipe.
propagating in a single mode is shown in Table 7.5. DC\textsubscript{1} and DC\textsubscript{3} both had 1:3 splitting ratios, while the splitting ratio of DC\textsubscript{2} was 50:50.

**Integrated Circuits experiments setup**

The experiments setup used for circuits with multi input/output ports, e.g., Circuit in subsection 7.13 and subsection 7.6 is consists of two 6-axis precision tables MAX603D (X,Y,Z, \(\theta\)x Pitch, \(\theta\)y Yaw, and \(\theta\)z, Roll). This setup is more adapted for measurements and coupling where several waveguides and arrays of fibers are butt coupled, which need more careful alignment in connection that transversal line of fibers in the array has to be aligned to several waveguides in inscribed circuits. Each an experiment sample it contained an alignment waveguides and if needed a calibration circuit.

**Figure 7.15:** Experiments fiber arrays grooves set up (left). Fiber array grooves set up alignment, lossy waveguides used for circuits position alignment (right).

I demonstrated optical photonic waveguide structures and interferometers in chip. By fabricating thermo-resistive elements tunable phase shifters were made. Beam splitters were also fabricated. The inherent stability of the in-chip circuits was demonstrated by building waveguide interferometers. These components and techniques may allow large photonic circuits to be constructed.
8. Photonic topological insulator by fs-laser written waveguides

Here in this chapter, I describe our research applying fs-laser writing capability to write in 2D and 3D array of waveguides to study topological photonic insulator systems.

8.1 Introduction

Over the last few years, arrays of evanescently coupled waveguides have been brought into focus as a particular representation of fictionalized optical materials, in which the dispersion and diffraction of propagating light can be specifically tuned. Moreover, it turns out that the light evolution in these systems shares fundamental similarities to the quantum evolution of particle wave-functions so that waveguides arrays can act as a model system for emulating quantum mechanics.

Topology as a property of a photonic material that characterizes the quantized behaviour of the wavefunctions on its entire dispersion band has emerged as another degree of freedom [79], which opens a path for finding new fundamental states of light and new applications. Since the discovery of topological insulators interest in various two and three-dimensional systems has greatly increased [80–83]. Recently researchers demonstrated photonic topological insulators in glass using fs-laser writing techniques [84; 85].

8.2 Topological insulator (TI)

In mathematics, the topological field is concerned with space global properties that are preserved under continuous deformations, e.g., bending. All the geometries that we are able to make by a bending and stretching an object without tearing it apart or glueing two regions together, they are topological equivalent, e.g., a coffee mug is topologically equivalent to a doughnut. A so-called topological insulator is a structure that is an insulator in its bulk but
Chapter 8. Photonic topological insulator by fs-laser written waveguides contains surface states that are conducting (metallic). They conduct electricity on their surface having no dissipation or back-scattering, even if there is a large presence of impurities.

8.3 Topological phases

A system can be in an ordered state at low temperature when it spontaneously loses one of the symmetries that is present at high temperature, e.g., crystals break its translational and rotational symmetries of free space. The topological phases (states) of the matter is one of the revolutionary discoveries in condensed matter physics, that cannot be explained by the symmetry breaking theory, which is used to describe most familiar phases of matter and the continuous phases transition. Quantum spin Hall phase is an example of a topological phase.

In quantum Hall system, the conductivity is perpendicular to the direction of the electric field, when confining electrons to 2D in low temperature and a strong perpendicular magnet is quantized (forming a cyclotron of discrete orbits energies) as:

\[ \sigma_{xy} = N \frac{e^2}{h}. \]  (8.1)

where \( N \equiv \) is an integer number called topological invariant, quantity that remains constant under continuous deformation of a system.

**Topological invariant genus number:**

In condensed matter physics, it has been discovered that it is possible to realize certain phases of matter (with unique physical properties) that can describe by the existence of a quantity named the topological invariant [86; 87] topological insulator is one of those. Topological invariant so-called genus \( (g) \) is a number which corresponds to the number of holes within a closed surface, see Fig

(a) \( g = 0 \)

(b) \( g = 1 \)

**Figure 8.1:** Topological two dimensional surfaces. (a) A sphere has genus \( g = 0 \). (b) A tour has genus \( g = 1 \).
8.3. Topological phases

8.1. The object only does a topological phase change if a hole is added or removed. The relation (Gauss-Bonnet theorem) which connects the geometry to topology can be written [79]:

\[
\frac{1}{2\pi} \int_{\text{surface}} k dA = 2(1 - g),
\] (8.2)

where \( k \) is the total Gaussian curvature of a two-dimensional closed surface. In the topological insulator the Berry phase plays a role to curvature. Integrating the Berry curvature over the torus surface gives a topological invariant called Chern number, it is a measure of the total quantized Berry flux on this two-dimensional closed surface.

The topological states can only be changed through a quantum phase transition, a process when the bulk gap needs to be closed, i.e., have gap-less states on its surface (metallic). Fig 8.2 (c) [88] illustrates the electronic dispersions of the valence sub-band and the conduction sub-band, dependent of the thicknesses of the quantum well.

A so-called quantum spin Hall phase, is another class of topological insulators in two dimensions, in which opposite spins move in opposite directions on the edge [80; 89; 90], the electrons in any given frame of spin-up must go one direction while the spin-down go the other direction and will not feel each other at all. This model contains two edge states propagating in the opposite direction and with opposite spin, in which the time-reversal (T) is broken by the magnetic field. The topological insulator states come as the results of materials with a Dirac point (Dirac point is a location in the band structure where two bands of some material having two cones like tips meet at a point.) somewhere in its band structure. Dirac cones are protected, by parity inversion and time-reversal (PT) symmetry, see Fig.8.4 (top left) a band diagram of edge

![Figure 8.2: Two-dimensional band structure of a HgTe quantum well depending on its thickness (d).](image-url)

(a) When \( d < d_c \), the system is a trivial insulator, (b) when \( d = d_c \), the band gap is closed and the system is metallic, (c) when \( d > d_c \), the system is a two-dimensional topological insulator [88].
states where the bulk dispersions form a pair of Dirac cones in protected by PT symmetry. The quantum Hall phase is a T-breaking phase of non-zero Chern numbers in red in the phase diagram for the same Fig.8.4 [79].

8.4 Topological photonics

The model (quantum Hall effect) was transferred theoretically to the field of photonics in crystal by Haldane and Raghu [91; 92]. The idea was confirmed and experimentally observed, and numerous studies were followed [83; 93–
The key feature of the quantum anomalous Hall effect was transferred to the realm of photonics \[94\] by applying a uniform magnetic field to gyromagnetic photonic crystals making the first realization in a photonic analog of the quantum Hall effect at microwave frequencies. By breaking the T symmetry when applying a uniform magnetic field on gyromagnetic photonic crystals, it results in a single topologically protected edge mode showed a propagation around arbitrary disorder without reflection. The topological phases in electronic materials are related to the wave-particle duality of the electron. Since the crystal translation symmetry always exits, one can calculate the band structures from the equation:

\[
\mathcal{H}(k)|u_n(k)\rangle = E_n(k)|u_n(k)\rangle ,
\]

where \(\mathcal{H}(k)\) is the Bloch Hamiltonian, \(u_n(k)\) is the Bloch wave function, and \(E_n(k)\) is the Bloch eigenvalue, while \(k\) is the crystal momentum which is well defined. As \(k\) goes around the Brillouin zone, the Bloch wave function acquires Berry phase geometry. It is convenient to classify the topological phases with respect to symmetries, e.g., topological gapped phases, topological gap-less phases, and higher-order topological phases.

Recently, higher-order topological insulators used to describe systems with lower dimension gap-less boundary states \[82; 99–106\]. Higher-order topological insulators are having non-trivial topological insulating phases. The \(n\)th order of the topological insulator (TI) is defined as a \(dD\) T , with \((d-1)D\), \((d-2)D\), ..., \((d-n)D\) gapped boundary states and \((d-n)D\) gap-less boundary states, e.g., the 0D boundary states is gap-less which are called corner states light.

### Propagation of light

The photonic material property that characterizes the global quantized behaviour of the wavefunction on its entire dispersion band namely topology opens the possibility for new states of light. In these systems, Maxwell’s equations describing the propagation of light amounts to the paraxial as:

\[
\nabla \times \nabla \times E = \varepsilon \left(\frac{\omega}{c}\right)^2 E,
\]

\(\varepsilon\) \(\equiv\) dielectric constant function in \((x,y,z)\), \(E\) is the electric field, \(\omega\) is the light frequency and \(c\) is the speed of light. Assuming that \(\varepsilon(x,y,z) = \varepsilon_o + \Delta \varepsilon(x,y,z)\) (small variation) one can write:

\[
-\nabla^2 E = \varepsilon \left(\frac{\omega}{c}\right)^2 E + \nabla(E \cdot \frac{\nabla \varepsilon}{\varepsilon}),
\]
the value $\varepsilon_0 = n_o^2$ where $n_o$ is the refractive index, and the wavelength in the medium $\lambda_o = \frac{2\pi}{k_o}$ which is equal to $\frac{2\pi c}{n_o \omega}$ assuming the scale of variation is larger than the wavelength. When the light propagates along the z-directional of the waveguides axis the wave-vector ($k_o = K_z \gg k_{x,y}$). The electric field in the carrier-envelope can be written as:

$$E(x,y,z) = \hat{e} \tilde{E}(x,y,z) \exp(ik_o z),$$

(8.6)

where $\hat{e}$ is the unit vector and $\tilde{E}$ the varying function satisfying $|\nabla \tilde{E}| \ll |k_o \tilde{E}|$. The propagation equation then can be written as:

$$-\partial_z^2 \tilde{E} - 2ik_o \partial_z \tilde{E} - \nabla^2 \tilde{E} + k_o^2 \tilde{E} = \varepsilon(\frac{\omega}{c})^2 \tilde{E},$$

(8.7)

where $\nabla_\perp$ is an operator that acts on the transverse x,y-plane. Assuming $\tilde{E}$ is varying slowly in z-direction, then the term $\partial_z^2 \tilde{E}$ will be neglected and the paraxial equation of the diffraction of light can be written as:

$$i \partial_z \tilde{E} = -\frac{1}{2k_o} \nabla^2 \tilde{E} - \frac{k_o \triangle n}{n_o} \tilde{E},$$

(8.8)

where the convenience refractive index $n = \sqrt{\varepsilon} = n_o + \triangle n \cong n_o + \triangle \varepsilon/(2n_o)$. The equation when the field is slowly varying in strong confinement within waveguides, and using the tight-binding approximation, one can find that the light propagating inside the waveguide is determined by the index $n_o$ as well as lateral confinement length [107]. The propagation of light can have non-trivial temporal dynamics, so that the time $t$ is like a third spatial coordinate, an additional kinetic energy term then with respect to the temporal direction will be added [108] and the propagating paraxial equation can take the form:

$$i \frac{\partial \tilde{E}}{\partial z} = -\frac{1}{2k_o} \nabla^2 \tilde{E} - \frac{k_o \triangle n}{n_o} \tilde{E} - \frac{1}{2m_t} \frac{\partial \tilde{E}}{\partial t^2},$$

(8.9)

where the coefficient $m_t^{-1} \equiv -\frac{\partial^2 k}{\partial \omega^2}$ is proportional to the group velocity in the medium. One can see the formal similarity of the equation and the Schrödinger equation.

### 8.5 Waveguides structures design and fabrication

The topological photonic waveguides circuits are fabricated using femtosecond laser (BlueCut femtosecond laser from Menlo Systems). The waveguides are written in Corning EAGLE2000 alumino-borosilicate glass substrate of 50 mm
length. The femtosecond laser produces pulses centred at a wavelength of 1030 μm, having a duration of 350 fs, and a repetition rate of 1 MHz. To inscribe the three-dimensional waveguides, pulses of 210 nJ were focused at depths (according to the circuits designed) of between 70 to 175 μm under the sample surface using a 50 × objective with a numerical aperture (NA) of 0.55, while the sample was translated at a constant speed of 30 mm s⁻¹ by a high-precision three axes translation stage (A3200, Aerotech Inc.). After the WG structure was written in the glass sample, the facets were carefully polished down to the optical quality of 0.1 μm. The facets were characterized with help of a Dino microscope, where the real obtained position of the WG structure in the glass sample could be established. The fabricated waveguides support single-mode at 780 nm as an output, having mode diameter (1/e²) of approximately from 6 – 8 μm. Refractive index increase of about 3.9 × 10⁻³. The propagation losses were estimated to be around 0.3 dB/cm and the birefringence in the order of 7 × 10⁻⁵.

8.6 Experimental setup

The experimental setup used for topological measurements as in the Fig 8.5 used to characterize the outputs from the array structures used in the study. The waveguides (WGs) were excited using a tunable wavelengths laser in visible and NIR (Cameleon Ultra II, Coherent). H-polarized light and a focusing system at the input allow exciting each WG separately. At the output, a collecting lens and a camera (CCD) were used to measure the intensity of out coming light from different waveguides.

For the observation of the topological isolation of the waveguides structure,

![Figure 8.5: The setup used to characterize and perform the topological photonic circuits measurements.](image-url)

the beam from a tunable Infra-red laser was launched into the system (the experiment scanning range inside the single mode regime governed by the V...
Chapter 8. Photonic topological insulator by fs-laser written waveguides

parameter.) using a 100× objective of 0.9 NA, which was sufficient for single excitation of each waveguide composing the photonic structure. The output light of the glass substrate was collected with help of a 100× high NA objective and a camera that allows to get the picture images profile from all the waveguides forming the topological structure.

8.7 Su-Schrieffer-Heeger (SSH) model

One of the most elegant examples of a topological system is the one-dimensional Su-Schrieffer-Heeger (SSH) chain, which consists of two sites in the unit cell and features robust zero-energy modes.

The SSH model describes a one-dimensional chain in which only nearest-neighbour hopping terms are allowed. The model can be described by the Hamiltonian:

\[ H = \sum_{i=1}^{N} (t_1 c_{i-1}^\dagger c_i + t_2 c_{i+1}^\dagger c_i + h.c.), \] (8.10)

where \( t_1 \) and \( t_2 \) are nearest-neighbour hopping parameters, \( c_{i}^\dagger \) and \( c_{i} \) creates and annihilates photon at site \( i \), while \( h.c. \), the hermitian conjugate. The Hamiltonian for open boundary condition (\( H = \psi^\dagger \mathcal{H} \psi \)) can be written as:

\[
\mathcal{H} = \begin{bmatrix}
0 & t_1 & 0 & 0 & \cdots \\
t_1 & 0 & t_2 & t_1 & 0 & \cdots \\
t_2 & 0 & t_1 & t_2 & \ddots & \ddots \\
t_1 & 0 & t_2 & 0 & \ddots & \ddots \\
t_2 & 0 & \ddots & \ddots & \ddots & \ddots \\
0 & \ddots & \ddots & \ddots & \ddots & \ddots \\
\end{bmatrix}
\] (8.11)

The exact eigenstate of the Hamiltonian is then:

\[
|\psi\rangle = N \sum_{n=0}^{N-1} \left(-\frac{t_1}{t_2}\right)^n c_{A,n}^\dagger |0\rangle,
\] (8.12)

where \( c_{A}^\dagger \) creates an electron on sublattice A site n. To realize the SSH model using photons, hopping models can be directly translated to these experimental realizations by using the fact that the hopping parameter \( t \) in the theoretical model scales with the distance \( d \) between two waveguides according to \( t \sim e^{-d/\xi} \) with \( \xi \) parameter set experimentally related to the dimensions of length. Series of waveguides have been inscribed using Fs laser see table 8.1.
The close distances separation between the waveguides were carried out to determine the evanescent coupling between them. The SSH chain is a symmetry-protected topological system and consists of a one-dimensional tight-binding hopping model with alternating hopping strengths $t_1$ and $t_2$.

<table>
<thead>
<tr>
<th>Fabrication parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass type</td>
</tr>
<tr>
<td>Pulse repetition rate</td>
</tr>
<tr>
<td>Objective</td>
</tr>
<tr>
<td>Writing wavelength</td>
</tr>
<tr>
<td>Writing speed</td>
</tr>
<tr>
<td>Pulse energy</td>
</tr>
</tbody>
</table>

**Table 8.1:** 1D SSH model circuit fabrication recipe.

![Figure 8.6: SSH model for 11 waveguides. (a) The coupling ratio $\frac{d_1}{d_2} = 1$. (b) The coupling ratio $\frac{d_1}{d_2} = 0.5$. (c) The coupling ratio $\frac{d_1}{d_2} = 1.5$.](image)

We also studied two more complicated systems using 3D structures, i.e., networks of models having corner-sharing, rhombic and triangular geometries. Each of the edges of both lattices has the exact same structure as the SSH chain, the design imposes that only neighbouring waveguides in the structure can interact (similarly to the 1D chain SSH model) such that they conspire to generate the appearance of zero-energy modes at the corners. The rhombus structure designed with an odd number of sites in each edge and the triangle structure designed with an even number of sites in each edge.

### 8.8 Experimental results

Many circuits were fabricated for different ratios, i.e., $d_1$ and $d_2$ corresponding to the hopping parameters $t_1$ and $t_2$ in the Hamiltonian. The coupling strength between the guides was determined in preliminary experiments, i.e., distances
Figure 8.7: Simulation outputs of (a) The band spectrum of the SSH model as a function of the model parameter $d_1$ and $d_2$, the vertical lines indicate the ratio $\frac{d_1}{d_2}$. (b) The output intensity for the ratio $\frac{d_1}{d_2} = 0.5$ for 11 waveguides. (c) The output intensity for the ratio $\frac{d_1}{d_2} = 1$ for 11 waveguides. (d) The output intensity for the ratio $\frac{d_1}{d_2} = 1.5$ for 11 waveguides.

vary from 5 to 30 $\mu$m in a gradient of 2 $\mu$m have studied as a function of inter-site spacing and interaction length the results are reported in Fig. 8.8.

By setting $d_1 = 12 \mu$m and $d_2 = 7 \mu$m, the experiments which is limit of well localised light where $t_1 < t_2$ and all other terms negligible. The results for the 1D SSH model studied are shown in Fig. 8.9, Fig.8.10, and Fig.8.11, the designed structures; used to study the stability of the corner state concerning different injection positions are reported in Paper [IV].

8.9 Summary and Conclusion

We demonstrated experimentally topological photonic structures in one dimensional arrays of waveguides. The waveguides photonic structure inscribed by femtosecond laser writing in a glass. We realized 3D lattice structures of a rhombus and a triangle experimentally using photonic structures fabricated in borosilicate glass. Optical waveguides have provided an extremely rich laboratory tool to visualize with optical waves the classic analogy of a wide variety of
8.9. Summary and Conclusion

Figure 8.8: Normalized output dependants on separation distances between the straight waveguides.

Figure 8.9: The CCD camera output results of SSH 1D with $d_1/d_2 = 1$ and 50 mm waveguides length. The result can be achieved no matter the excitation takes place from which far side of the 11 waveguides series.

Figure 8.10: The CCD camera output results of SSH 1D with $d_1/d_2 = 0.65$ and 50 mm waveguides length. 11 waveguides in series the red circle shows the position of excited waveguide.
coherent quantum phenomena encountered in atomic, molecular or condensed-matter physics. The methods used offers the advantage of direct mapping of the wave function evolution in coordinate space by simple structures designs similar to the spatial light propagation along with the guiding structure, which can be precisely mapped.
References


[68] Corning. This is a test entry of type @ONLINE, February 2019. 59


References


Part III

Scientific Publications